1. You are given the following information about a stationary AR(2) model:

(i) $\rho_1 = 0.5$

(ii) $\rho_2 = 0.1$

Determine $\phi_2$.

(A) –0.2

(B) 0.1

(C) 0.4

(D) 0.7

(E) 1.0
2. You are given:

(i) Losses follow a loglogistic distribution with cumulative distribution function:

\[ F(x) = \frac{(x / \theta)^\gamma}{1 + (x / \theta)^\gamma} \]

(ii) The sample of losses is:

10 35 80 86 90 120 158 180 200 210 1500

Calculate the estimate of \( \theta \) by percentile matching, using the 40\(^{th}\) and 80\(^{th}\) empirically smoothed percentile estimates.

(A) Less than 77
(B) At least 77, but less than 87
(C) At least 87, but less than 97
(D) At least 97, but less than 107
(E) At least 107
3. You are given:

(i) The number of claims has a Poisson distribution.

(ii) Claim sizes have a Pareto distribution with parameters $\theta = 0.5$ and $\alpha = 6$.

(iii) The number of claims and claim sizes are independent.

(iv) The observed pure premium should be within 2% of the expected pure premium 90% of the time.

Determine the expected number of claims needed for full credibility.

(A) Less than 7,000

(B) At least 7,000, but less than 10,000

(C) At least 10,000, but less than 13,000

(D) At least 13,000, but less than 16,000

(E) At least 16,000
4. You study five lives to estimate the time from the onset of a disease to death. The times to death are:

\[2\quad 3\quad 3\quad 3\quad 7\]

Using a triangular kernel with bandwidth 2, estimate the density function at 2.5.

(A) \(8/40\)
(B) \(12/40\)
(C) \(14/40\)
(D) \(16/40\)
(E) \(17/40\)
5. For the model $Y_i = \alpha + \beta X_i + \varepsilon_i$, where $i = 1,2,\ldots,10$, you are given:

(i) $X_i = \begin{cases} 1, & \text{if the } i\text{th individual belongs to a specified group} \\ 0, & \text{otherwise} \end{cases}$

(ii) 40 percent of the individuals belong to the specified group.

(iii) The least squares estimate of $\beta$ is $\hat{\beta} = 4$.

(iv) $\sum (Y_i - \hat{\alpha} - \hat{\beta} X_i)^2 = 92$

Calculate the $t$ statistic for testing $H_0: \beta = 0$.

(A) 0.9

(B) 1.2

(C) 1.5

(D) 1.8

(E) 2.1
6. You are given:

(i) Losses follow a Single-parameter Pareto distribution with density function:

\[ f(x) = \frac{\alpha}{x^{\alpha+1}}, \quad x > 1, \quad 0 < \alpha < \infty \]

(ii) A random sample of size five produced three losses with values 3, 6 and 14, and two losses exceeding 25.

Determine the maximum likelihood estimate of \( \alpha \).

(A) 0.25
(B) 0.30
(C) 0.34
(D) 0.38
(E) 0.42
7. You are given:

(i) The annual number of claims for a policyholder has a binomial distribution with probability function:

\[ p(x | q) = \binom{2}{x} q^x (1-q)^{2-x}, \quad x = 0, 1, 2 \]

(ii) The prior distribution is:

\[ \pi(q) = 4q^3, \quad 0 < q < 1 \]

This policyholder had one claim in each of Years 1 and 2.

Determine the Bayesian estimate of the number of claims in Year 3.

(A) Less than 1.1
(B) At least 1.1, but less than 1.3
(C) At least 1.3, but less than 1.5
(D) At least 1.5, but less than 1.7
(E) At least 1.7
8. For a sample of dental claims $x_1, x_2, \ldots, x_{10}$, you are given:

(i) $\sum x_i = 3860$ and $\sum x_i^2 = 4,574,802$

(ii) Claims are assumed to follow a lognormal distribution with parameters $\mu$ and $\sigma$.

(iii) $\mu$ and $\sigma$ are estimated using the method of moments.

Calculate $E[X \wedge 500]$ for the fitted distribution.

(A) Less than 125

(B) At least 125, but less than 175

(C) At least 175, but less than 225

(D) At least 225, but less than 275

(E) At least 275
9. You are given:

(i) \( Y_{ij} \) is the loss for the \( j \)th insured in the \( i \)th group in Year \( t \).

(ii) \( \bar{Y}_{ij} \) is the mean loss in the \( i \)th group in Year \( t \).

(iii) \( X_{ij} = \begin{cases} 0, & \text{if the \( j \)th insured is in the first group (\( i = 1 \))} \\ 1, & \text{if the \( j \)th insured is in the second group (\( i = 2 \))} \end{cases} \)

(iv) \( Y_{2ij} = \delta + \phi Y_{ij} + \theta X_{ij} + \epsilon_{ij} \), where \( i = 1,2 \) and \( j = 1,2,...,n \)

(v) \( \bar{Y}_{21} = 30, \bar{Y}_{22} = 37, \bar{Y}_{11} = 40, \bar{Y}_{12} = 41 \)

(vi) \( \hat{\phi} = 0.75 \)

Determine the least-squares estimate of \( \theta \).

(A) 5.25

(B) 5.50

(C) 5.75

(D) 6.00

(E) 6.25
10. Two independent samples are combined yielding the following ranks:

Sample I: 1, 2, 3, 4, 7, 9, 13, 19, 20
Sample II: 5, 6, 8, 10, 11, 12, 14, 15, 16, 17, 18

You test the null hypothesis that the two samples are from the same continuous distribution.

The variance of the rank sum statistic is:

\[ \frac{nm(n + m + 1)}{12} \]

Using the classical approximation for the two-tailed rank sum test, determine the \( p \)-value.

(A) 0.015
(B) 0.021
(C) 0.105
(D) 0.210
(E) 0.420
11. You are given:

(i) Claim counts follow a Poisson distribution with mean $\theta$.

(ii) Claim sizes follow an exponential distribution with mean $10\theta$.

(iii) Claim counts and claim sizes are independent, given $\theta$.

(iv) The prior distribution has probability density function:

$$\pi(\theta) = \frac{5}{\theta^6}, \quad \theta > 1$$

Calculate Bühlmann’s $k$ for aggregate losses.

(A) Less than 1

(B) At least 1, but less than 2

(C) At least 2, but less than 3

(D) At least 3, but less than 4

(E) At least 4
12. You are given:

(i) A survival study uses a Cox proportional hazards model with covariates $Z_1$ and $Z_2$, each taking the value 0 or 1.

(ii) The maximum partial likelihood estimate of the coefficient vector is:

$$\left(\hat{\beta}_1, \hat{\beta}_2\right) = (0.71, 0.20)$$

(iii) The baseline survival function at time $t_0$ is estimated as $\hat{S}(t_0) = 0.65$.

Estimate $S(t_0)$ for a subject with covariate values $Z_1 = Z_2 = 1$.

(A) 0.34
(B) 0.49
(C) 0.65
(D) 0.74
(E) 0.84
13. You are given:

(i) $Z_1$ and $Z_2$ are independent $N(0,1)$ random variables.

(ii) $a, b, c, d, e, f$ are constants.

(iii) $Y = a + bZ_1 + cZ_2$ and $X = d + eZ_1 + fZ_2$

Determine $E(Y | X)$.

(A) $a$

(B) $a + (b + c)(X - d)$

(C) $a + (be + cf)(X - d)$

(D) $a + \left[\frac{(be + cf)}{(e^2 + f^2)}\right]X$

(E) $a + \left[\frac{(be + cf)}{(e^2 + f^2)}\right](X - d)$
14. You are given:

(i) Losses on a company’s insurance policies follow a Pareto distribution with probability density function:

\[ f(x|\theta) = \frac{\theta}{(x + \theta)^2}, \quad 0 < x < \infty \]

(ii) For half of the company’s policies, \( \theta = 1 \), while for the other half, \( \theta = 3 \).

For a randomly selected policy, losses in Year 1 were 5.

Determine the posterior probability that losses for this policy in Year 2 will exceed 8.

(A) 0.11

(B) 0.15

(C) 0.19

(D) 0.21

(E) 0.27
15. You are given total claims for two policyholders:

<table>
<thead>
<tr>
<th>Policyholder</th>
<th>Year</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td></td>
<td>730</td>
<td>800</td>
<td>650</td>
<td>700</td>
</tr>
<tr>
<td>Y</td>
<td></td>
<td>655</td>
<td>650</td>
<td>625</td>
<td>750</td>
</tr>
</tbody>
</table>

Using the nonparametric empirical Bayes method, determine the Bühlmann credibility premium for Policyholder Y.

(A) 655  
(B) 670  
(C) 687  
(D) 703  
(E) 719
16. A particular line of business has three types of claims. The historical probability and the number of claims for each type in the current year are:

<table>
<thead>
<tr>
<th>Type</th>
<th>Historical Probability</th>
<th>Number of Claims in Current Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.2744</td>
<td>112</td>
</tr>
<tr>
<td>B</td>
<td>0.3512</td>
<td>180</td>
</tr>
<tr>
<td>C</td>
<td>0.3744</td>
<td>138</td>
</tr>
</tbody>
</table>

You test the null hypothesis that the probability of each type of claim in the current year is the same as the historical probability.

Calculate the chi-square goodness-of-fit test statistic.

(A) Less than 9
(B) At least 9, but less than 10
(C) At least 10, but less than 11
(D) At least 11, but less than 12
(E) At least 12
17. Which of the following is false?

(A) If the characteristics of a stochastic process change over time, then the process is nonstationary.

(B) Representing a nonstationary time series by a simple algebraic model is often difficult.

(C) Differences of a homogeneous nonstationary time series will always be nonstationary.

(D) If a time series is stationary, then its mean, variance and, for any lag $k$, covariance must also be stationary.

(E) If the autocorrelation function for a time series is zero (or close to zero) for all lags $k > 0$, then no model can provide useful minimum mean-square-error forecasts of future values other than the mean.
18. The information associated with the maximum likelihood estimator of a parameter \( \theta \) is \( 4n \), where \( n \) is the number of observations.

Calculate the asymptotic variance of the maximum likelihood estimator of \( 2\theta \).

(A) \( \frac{1}{2n} \)

(B) \( \frac{1}{n} \)

(C) \( \frac{4}{n} \)

(D) \( 8n \)

(E) \( 16n \)
19. You are given:

(i) The probability that an insured will have at least one loss during any year is $p$.

(ii) The prior distribution for $p$ is uniform on $[0, 0.5]$.

(iii) An insured is observed for 8 years and has at least one loss every year.

Determine the posterior probability that the insured will have at least one loss during Year 9.

(A) 0.450
(B) 0.475
(C) 0.500
(D) 0.550
(E) 0.625
20. At the beginning of each of the past 5 years, an actuary has forecast the annual claims for a group of insureds. The table below shows the forecasts \((X)\) and the actual claims \((Y)\). A two-variable linear regression model is used to analyze the data.

<table>
<thead>
<tr>
<th>(t)</th>
<th>(X_i)</th>
<th>(Y_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>475</td>
<td>254</td>
</tr>
<tr>
<td>2</td>
<td>254</td>
<td>463</td>
</tr>
<tr>
<td>3</td>
<td>463</td>
<td>515</td>
</tr>
<tr>
<td>4</td>
<td>515</td>
<td>567</td>
</tr>
<tr>
<td>5</td>
<td>567</td>
<td>605</td>
</tr>
</tbody>
</table>

You are given:

(i) The null hypothesis is \(H_0: \alpha = 0, \beta = 1\).

(ii) The unrestricted model fit yields ESS = 69,843.

Which of the following is true regarding the \(F\) test of the null hypothesis?

(A) The null hypothesis is not rejected at the 0.05 significance level.

(B) The null hypothesis is rejected at the 0.05 significance level, but not at the 0.01 level.

(C) The numerator has 3 degrees of freedom.

(D) The denominator has 2 degrees of freedom.

(E) The \(F\) statistic cannot be determined from the information given.
21-22. Use the following information for questions 21 and 22.

For a survival study with censored and truncated data, you are given:

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Number at Risk at Time t</th>
<th>Failures at Time t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>4</td>
</tr>
</tbody>
</table>

21. The probability of failing at or before Time 4, given survival past Time 1, is \( \hat{q}_1 \).

Calculate Greenwood’s approximation of the variance of \( \hat{q}_1 \).

(A) 0.0067
(B) 0.0073
(C) 0.0080
(D) 0.0091
(E) 0.0105
21-22. *(Repeated for convenience)* Use the following information for questions 21 and 22.

For a survival study with censored and truncated data, you are given:

<table>
<thead>
<tr>
<th>Time ($t$)</th>
<th>Number at Risk at Time $t$</th>
<th>Failures at Time $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td>4</td>
</tr>
</tbody>
</table>

22. Calculate the 95% log-transformed confidence interval for $H(3)$, based on the Nelson-Aalen estimate.

(A) (0.30, 0.89)
(B) (0.31, 1.54)
(C) (0.39, 0.99)
(D) (0.44, 1.07)
(E) (0.56, 0.79)
23. You are given:

(i) Two risks have the following severity distributions:

<table>
<thead>
<tr>
<th>Amount of Claim</th>
<th>Probability of Claim Amount for Risk 1</th>
<th>Probability of Claim Amount for Risk 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>2,500</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>60,000</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(ii) Risk 1 is twice as likely to be observed as Risk 2.

A claim of 250 is observed.

Determine the Bühlmann credibility estimate of the second claim amount from the same risk.

(A) Less than 10,200

(B) At least 10,200, but less than 10,400

(C) At least 10,400, but less than 10,600

(D) At least 10,600, but less than 10,800

(E) At least 10,800
24. You are given:

(i) A sample \( x_1, x_2, \ldots, x_{10} \) is drawn from a distribution with probability density function:

\[
\frac{1}{2} \left[ \frac{1}{\theta} \exp\left(\frac{-x}{\theta}\right) + \frac{1}{\sigma} \exp\left(\frac{-x}{\sigma}\right) \right], \quad 0 < x < \infty
\]

(ii) \( \theta > \sigma \)

(iii) \( \sum x_i = 150 \) and \( \sum x_i^2 = 5000 \)

Estimate \( \theta \) by matching the first two sample moments to the corresponding population quantities.

(A) 9
(B) 10
(C) 15
(D) 20
(E) 21
25. You are given the following time-series model:

\[ y_t = 0.8 y_{t-1} + 2 + \varepsilon_t - 0.5 \varepsilon_{t-1} \]

Which of the following statements about this model is false?

(A) \( \rho_1 = 0.4 \)

(B) \( \rho_k < \rho_1, \ k = 2, 3, 4, \ldots \)

(C) The model is ARMA(1,1).

(D) The model is stationary.

(E) The mean, \( \mu \), is 2.
26. You are given a sample of two values, 5 and 9.

You estimate $\text{Var}(X)$ using the estimator $g(X_1, X_2) = \frac{1}{2} \sum (X_i - \bar{X})^2$.

Determine the bootstrap approximation to the mean square error of $g$.

(A) 1

(B) 2

(C) 4

(D) 8

(E) 16
27. You are given:

(i) The number of claims incurred in a month by any insured has a Poisson distribution with mean $\lambda$.

(ii) The claim frequencies of different insureds are independent.

(iii) The prior distribution is gamma with probability density function:

$$f(\lambda) = \frac{(100\lambda)^6 e^{-100\lambda}}{120\lambda}$$

(iv) The number of insureds and claims for each month:

<table>
<thead>
<tr>
<th>Month</th>
<th>Number of Insureds</th>
<th>Number of Claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>150</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>300</td>
<td>?</td>
</tr>
</tbody>
</table>

Determine the Bühlmann-Straub credibility estimate of the number of claims in Month 4.

(A) 16.7  
(B) 16.9  
(C) 17.3  
(D) 17.6  
(E) 18.0
28. You fit a Pareto distribution to a sample of 200 claim amounts and use the likelihood ratio test to test the hypothesis that $\alpha = 1.5$ and $\theta = 7.8$.

You are given:

(i) The maximum likelihood estimates are $\hat{\alpha} = 1.4$ and $\hat{\theta} = 7.6$.

(ii) The natural logarithm of the likelihood function evaluated at the maximum likelihood estimates is $-817.92$.

(iii) $\sum \ln(x_i + 7.8) = 607.64$

Determine the result of the test.

(A) Reject at the 0.005 significance level.

(B) Reject at the 0.010 significance level, but not at the 0.005 level.

(C) Reject at the 0.025 significance level, but not at the 0.010 level.

(D) Reject at the 0.050 significance level, but not at the 0.025 level.

(E) Do not reject at the 0.050 significance level.
29. You are given:

(i) The model is \( Y_i = \beta X_i + \varepsilon_i, \) \( i = 1, 2, 3. \)

(ii) 

<table>
<thead>
<tr>
<th>( i )</th>
<th>( X_i )</th>
<th>( \text{Var}(\varepsilon_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>16</td>
</tr>
</tbody>
</table>

(iii) The ordinary least squares residuals are \( \hat{\varepsilon}_i = Y_i - \hat{\beta} X_i, \) \( i = 1, 2, 3. \)

Determine \( E(\hat{\varepsilon}_1^2 | X_1, X_2, X_3). \)

(A) 1.0

(B) 1.8

(C) 2.7

(D) 3.7

(E) 7.6
30. For a sample of 15 losses, you are given:

(i) 

<table>
<thead>
<tr>
<th>Interval</th>
<th>Observed Number of Losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 2]</td>
<td>5</td>
</tr>
<tr>
<td>(2, 5]</td>
<td>5</td>
</tr>
<tr>
<td>(5, ∞)</td>
<td>5</td>
</tr>
</tbody>
</table>

(ii) Losses follow the uniform distribution on \((0, \theta)\).

Estimate \(\theta\) by minimizing the function \(\sum_{j=1}^{3} \left( \frac{E_j - O_j}{O_j} \right)^2\), where \(E_j\) is the expected number of losses in the \(j\)th interval and \(O_j\) is the observed number of losses in the \(j\)th interval.

(A) 6.0
(B) 6.4
(C) 6.8
(D) 7.2
(E) 7.6
31. You are given:

(i) The probability that an insured will have exactly one claim is $\theta$.

(ii) The prior distribution of $\theta$ has probability density function:

$$\pi(\theta) = \frac{3}{2}\sqrt{\theta}, \quad 0 < \theta < 1$$

A randomly chosen insured is observed to have exactly one claim.

Determine the posterior probability that $\theta$ is greater than 0.60.

(A) 0.54

(B) 0.58

(C) 0.63

(D) 0.67

(E) 0.72
32. The distribution of accidents for 84 randomly selected policies is as follows:

<table>
<thead>
<tr>
<th>Number of Accidents</th>
<th>Number of Policies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>84</td>
</tr>
</tbody>
</table>

Which of the following models best represents these data?

(A) Negative binomial
(B) Discrete uniform
(C) Poisson
(D) Binomial
(E) Either Poisson or Binomial
33. A time series $y_t$ follows an ARIMA(1,1,1) model with $\phi_1 = 0.7$, $\theta_1 = -0.3$ and $\sigma^2 = 1.0$.

Determine the variance of the forecast error two steps ahead.

(A) 1
(B) 5
(C) 8
(D) 10
(E) 12
34. You are given:

(i) Low-hazard risks have an exponential claim size distribution with mean $\theta$.
(ii) Medium-hazard risks have an exponential claim size distribution with mean $2\theta$.
(iii) High-hazard risks have an exponential claim size distribution with mean $3\theta$.
(iv) No claims from low-hazard risks are observed.
(v) Three claims from medium-hazard risks are observed, of sizes 1, 2 and 3.
(vi) One claim from a high-hazard risk is observed, of size 15.

Determine the maximum likelihood estimate of $\theta$.

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5
35. You are given:

(i) \( X_{\text{partial}} = \) pure premium calculated from partially credible data

(ii) \( \mu = \mathbb{E}[X_{\text{partial}}] \)

(iii) Fluctuations are limited to \( \pm k \mu \) of the mean with probability \( P \)

(iv) \( Z = \) credibility factor

Which of the following is equal to \( P \)?

(A) \( \Pr[\mu - k \mu \leq X_{\text{partial}} \leq \mu + k \mu] \)

(B) \( \Pr[Z \mu - k \leq Z X_{\text{partial}} \leq Z \mu + k] \)

(C) \( \Pr[Z \mu - \mu \leq Z X_{\text{partial}} \leq Z \mu + \mu] \)

(D) \( \Pr[1 - k \leq Z X_{\text{partial}} + (1 - Z) \mu \leq 1 + k] \)

(E) \( \Pr[\mu - k \mu \leq Z X_{\text{partial}} + (1 - Z) \mu \leq \mu + k \mu] \)
36. For the model \( Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + \epsilon_i \), you are given:

(i) \( N = 15 \)

(ii) \[
(X'X)^{-1} = \begin{bmatrix}
13.66 & -0.33 & 2.05 & -6.31 \\
-0.33 & 0.03 & 0.11 & 0.00 \\
2.05 & 0.11 & 2.14 & -2.52 \\
-6.31 & 0.00 & -2.52 & 4.32
\end{bmatrix}
\]

(iii) \( \text{ESS} = 282.82 \)

Calculate the standard error of \( \hat{\beta}_3 - \hat{\beta}_2 \).

(A) 6.4
(B) 6.8
(C) 7.1
(D) 7.5
(E) 7.8
37. You are given:

<table>
<thead>
<tr>
<th>Claim Size $(X)$</th>
<th>Number of Claims</th>
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<tbody>
<tr>
<td>$(0, 25]$</td>
<td>25</td>
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<tr>
<td>$(25, 50]$</td>
<td>28</td>
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<tr>
<td>$(50, 100]$</td>
<td>15</td>
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<tr>
<td>$(100, 200]$</td>
<td>6</td>
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</tbody>
</table>

Assume a uniform distribution of claim sizes within each interval.

Estimate $E(X^2) - E[(X \wedge 150)^2].$

(A) Less than 200
(B) At least 200, but less than 300
(C) At least 300, but less than 400
(D) At least 400, but less than 500
(E) At least 500
Which of the following statements about moving average models is false?

(A) Both unweighted and exponentially weighted moving average (EWMA) models can be used to forecast future values of a time series.

(B) Forecasts using unweighted moving average models are determined by applying equal weights to a specified number of past observations of the time series.

(C) Forecasts using EWMA models may not be true averages because the weights applied to the past observations do not necessarily sum to one.

(D) Forecasts using both unweighted and EWMA models are adaptive because they automatically adjust themselves to the most recently available data.

(E) Using an EWMA model, the two-period forecast is the same as the one-period forecast.
39. You are given:

(i) Each risk has at most one claim each year.

(ii) Type of Risk | Prior Probability | Annual Claim Probability
<table>
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<tbody>
<tr>
<td>I</td>
<td>0.7</td>
<td>0.1</td>
</tr>
<tr>
<td>II</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>III</td>
<td>0.1</td>
<td>0.4</td>
</tr>
</tbody>
</table>

One randomly chosen risk has three claims during Years 1-6.

Determine the posterior probability of a claim for this risk in Year 7.

(A) 0.22
(B) 0.28
(C) 0.33
(D) 0.40
(E) 0.46
40. You are given the following about 100 insurance policies in a study of time to policy surrender:

(i) The study was designed in such a way that for every policy that was surrendered, a new policy was added, meaning that the risk set, \( r_j \), is always equal to 100.

(ii) Policies are surrendered only at the end of a policy year.

(iii) The number of policies surrendered at the end of each policy year was observed to be:

1 at the end of the 1\textsuperscript{st} policy year
2 at the end of the 2\textsuperscript{nd} policy year
3 at the end of the 3\textsuperscript{rd} policy year
...
\( n \) at the end of the \( n \textsuperscript{th} \) policy year

(iv) The Nelson-Aalen empirical estimate of the cumulative distribution function at time \( n \), \( \hat{F}(n) \), is 0.542.

What is the value of \( n \)?

(A) 8
(B) 9
(C) 10
(D) 11
(E) 12

**END OF EXAMINATION**
## Course 4, Fall 2003

### PRELIMINARY ANSWER KEY

<table>
<thead>
<tr>
<th>Question #</th>
<th>Answer</th>
<th>Question #</th>
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<td>20</td>
<td>A</td>
<td>40</td>
<td>E</td>
</tr>
</tbody>
</table>
Question #1
Key: A

The Yule-Walker equations are:
\[ .5 = \phi_1 + .5\phi_2 \]
\[ .1 = .5\phi_1 + \phi_2 \]
The solution is \( \phi_2 = -.2 \).

Question #2
Key: E

The 40th percentile is the .4(12) = 4.8th smallest observation. By interpolation it is .2(86) + .8(90) = 89.2. The 80th percentile is the .8(12) = 9.6th smallest observation. By interpolation it is .4(200) + .6(210) = 206.

The equations to solve are
\[ .4 = \frac{(89.2 / \theta)^\gamma}{1+(89.2 / \theta)^\gamma} \quad \text{and} \quad .8 = \frac{(206 / \theta)^\gamma}{1+(206 / \theta)^\gamma}. \]

Solving each for the parenthetical expression gives \( \frac{2}{3} = (89.2 / \theta)^\gamma \) and \( 4 = (206 / \theta)^\gamma \).

Taking the ratio of the second equation to the first gives \( 6 = (206 / 89.2)^\gamma \) which leads to \( \gamma = \ln(6) / \ln(206 / 89.2) = 2.1407 \). Then \( 4^{1.1407} = 206 / \theta \) for \( \theta = 107.8 \).

Question #3
Key: E

The standard for full credibility is
\[ \left( \frac{1.645}{.02} \right)^2 \left( 1 + \frac{\text{Var}(X)}{E(X)^2} \right) \]
where \( X \) is the claim size variable.

For the Pareto variable, \( E(X) = .5 / 5 = .1 \) and \( \text{Var}(X) = \frac{2(.5)^2}{5(4)} - (.1)^2 = .015 \). Then the standard is
\[ \left( \frac{1.645}{.02} \right)^2 \left( 1 + \frac{.015}{.1^2} \right) = 16,913 \text{ claims.} \]
Question #4
Key: B

The kernel is a triangle with a base of 4 and a height at the middle of 0.5 (so the area is 1). The length of the base is twice the bandwidth. Any observation within 2 of 2.5 will contribute to the estimate. For the observation at 2, when the triangle is centered at 2, the height of the triangle at 2.5 is .375 (it is one-quarter the way from 2 to the end of the triangle at 4 and so the height is one-quarter the way from 0.5 to 0). Similarly the points at 3 are also 0.5 away and so the height of the associated triangle is also .375. Each triangle height is weighted by the empirical probability at the associated point. So the estimate at 2.5 is \((1/5)(3/8) + (3/5)(3/8) + (1/5)(0) = 12/40\).

Question #5
Key: D

The standard error is \(\sqrt{92/8}\). With 4 of the \(X\)s equal to 1 and 6 equal to 0 the average is 0.4. Then

\[
\hat{\beta} = \frac{\sum (X_i - \bar{X})^2}{s^2} = \frac{92/8}{4(1-.4)^2 + 6(0-.4)^2} = 4.7917 \quad \text{and} \quad t = \frac{\hat{\beta}}{s} = \frac{4}{\sqrt{4.7917}} = 1.83.
\]

Question #6
Key: A

The distribution function is \(F(x) = \int x \alpha^{-\alpha-1} dt = -t^{-\alpha}\bigg|_0^\infty = 1 - x^{-\alpha}\). The likelihood function is

\[
L = f(3)f(6)f(14)[1 - F(25)]^2
= \alpha 3^{-\alpha-1} \alpha 6^{-\alpha-1} \alpha 14^{-\alpha-1} (25^{-\alpha})
\propto \alpha^3 (6(14)(625))^{-\alpha}.
\]

Taking logs, differentiating, setting equal to zero, and solving:

\[
\ln L = 3 \ln \alpha - \alpha \ln 157,500 \quad \text{plus a constant}
\]

\[
(\ln L)' = 3 \alpha^{-1} - \ln 157,500 = 0
\]

\[
\hat{\alpha} = 3 / \ln 157,500 = .2507.
\]
Question #7
Key: C

\[ \pi(q \mid 1,1) \propto p(1 \mid q)p(1 \mid q)\pi(q) = 2q(1-q)2q(1-q)4q^3 \propto q^5(1-q)^2 \]

\[ \int_0^1 q^5(1-q)^2 \, dq = \frac{1}{168}, \quad \pi(1,1) = 168q^2(1-q)^2. \]

The expected number of claims in a year is \( E(X \mid q) = 2q \) and so the Bayesian estimate is

\[ E(2q \mid 1,1) = \int_0^1 2q(168)q^5(1-q)^2 \, dq = \frac{4}{3}. \]

The answer can be obtained without integrals by recognizing that the posterior distribution of \( q \)

is beta with \( a = 6 \) and \( b = 3 \). The posterior mean is \( E(q \mid 1,1) = a/(a+b) = 6/9 = 2/3 \). The posterior mean of \( 2q \) is then \( 4/3 \).

Question #8
Key: D

For the method of moments estimate,

\[ 386 = e^{\mu + 5\sigma^2}, \quad 457,480.2 = e^{2\mu + 2\sigma^2} \]

\[ 5.9558 = \mu + 0.5\sigma^2, \quad 13.0335 = 2\mu + 2\sigma^2 \]

\[ \hat{\mu} = 5.3949, \quad \hat{\sigma}^2 = 1.1218. \]

Then

\[ E(X \land 500) = e^{5.3949 + 5(1.1218)} \Phi\left( \frac{\ln 500 - 5.3949 - 1.1218}{\sqrt{1.1218}} \right) + 500 \left[ 1 - \Phi\left( \frac{\ln 500 - 5.3949}{\sqrt{1.1218}} \right) \right] \]

\[ = 386\Phi(-.2853) + 500[1 - \Phi(.7739)] \]

\[ = 386(.3877) + 500(.2195) = 259. \]

Note-these calculations use exact normal probabilities. Rounding and using the normal table that accompanies the exam will produce a different numerical answer but the same letter answer.

Question #9
Key: E

Summing over \( i \) and \( j \) the least-squares quantity to minimize is

\[ \sum_{i=1}^2 \sum_{j=1}^n (Y_{2ij} - \delta - \phi Y_{1ij} - \theta X_j)^2 = \sum_{j=1}^n (Y_{11j} - \delta - \phi Y_{11j})^2 + \sum_{j=1}^n (Y_{22j} - \delta - \phi Y_{12j} - \theta)^2 \]

where the sum is split for \( i=1 \) and \( i=2 \) where the \( X \) values are known. Differentiating with respect to the two variables gives the equations

\[ -2(30n - \delta n - .75(40)n) - 2(37n - \delta n - .75(41)n - \theta n) = 0 \]

\[ -2(37n - \delta n - .75(41)n - \theta n) = 0. \]

Substituting the second equation into the first implies that \( \delta = 0 \). Then the second equation yields \( \theta = 37 -.75(41) = 6.25 \).
Question #10
Key: D

Because the values are already ranked, the test statistic is immediately calculated as the sum of the given values for Sample I: \( R = 1 + 2 + 3 + 4 + 7 + 9 + 13 + 19 + 20 = 78 \). The other needed values are \( n = 9 \) and \( m = 11 \), the two sample sizes. The mean is \( n(n+m+1)/2 = 94.5 \) and the variance is \( nm(n+m+1)/12 = 173.25 \). The test statistic is \( Z = (78 - 94.5) / \sqrt{173.25} = -1.25 \). The \( p \)-value is twice (because it is a two-tailed test) the probability of being more extreme than the test statistic, \( p = 2 \Pr(Z < -1.25) = .210 \).

Question #11
Key: C

Let \( N \) be the Poisson claim count variable, let \( X \) be the claim size variable, and let \( S \) be the aggregate loss variable.

\[
\begin{align*}
\mu(\theta) &= E(S \mid \theta) = E(N \mid \theta)E(X \mid \theta) = \theta 10\theta = 10\theta^2 \\
n(\theta) &= Var(S \mid \theta) = E(N \mid \theta)E(X^2 \mid \theta) = \theta 200\theta^2 = 200\theta^3 \\
\mu &= E(10\theta^2) = \int_1^{\infty} 10\theta^2 (5\theta^{-6}) d\theta = 50/3 \\
EPV &= E(200\theta^3) = \int_1^{\infty} 200\theta^3 (5\theta^{-6}) d\theta = 500 \\
VHM &= Var(10\theta^2) = \int_1^{\infty} (10\theta^2)^2 (5\theta^{-6}) d\theta - (50/3)^2 = 222.22 \\
k &= 500 / 222.22 = 2.25.
\end{align*}
\]

Question #12
Key: A

\( c = \exp(.71(1) + .20(1)) = 2.4843 \). Then \( \hat{S}(t_0; z) = \hat{S}_0(t_0)^c = .65^{2.4843} = .343 \).

Question #13
Key: E

\( Y \) and \( X \) are linear combinations of the same two normal random variables, so they are bivariate normal. Thus \( E(Y \mid X) = E(Y) + [\text{Cov}(Y,X)/\text{Var}(X)][X - E(X)] \). From the definitions of \( Y \) and \( X \), \( E(Y) = a \), \( E(X) = d \), \( \text{Var}(X) = e^2 + f^2 \), and \( \text{Cov}(Y,X) = be + cf \).
Question #14
Key: D

\[ \Pr(\theta = 1 \mid X = 5) = \frac{f(5 \mid \theta = 1) \Pr(\theta = 1)}{f(5 \mid \theta = 1) \Pr(\theta = 1) + f(5 \mid \theta = 3) \Pr(\theta = 3)} \]

\[ = \frac{(1/36)(1/2)}{(1/36)(1/2) + (3/64)(1/2)} = 16/43 \]

\[ \Pr(X_2 > 8 \mid X_1 = 5) = \Pr(X_2 > 8 \mid \theta = 1) \Pr(\theta = 1 \mid X_1 = 5) + \Pr(X_2 > 8 \mid \theta = 3) \Pr(\theta = 3 \mid X_1 = 5) \]

\[ = (1/9)(16/43) + (3/11)(27/43) = .2126. \]

For the last line, \( \Pr(X > 8 \mid \theta) = \int_8^{\infty} \theta(\theta + \theta^{-1}) d\theta = \theta(8 + \theta)^{-1} \) is used.

Question #15
Key: C

The sample mean for \( X \) is 720 and for \( Y \) is 670. The mean of all 8 observations is 695.

\[ (730 - 720)^2 + (800 - 720)^2 + (650 - 720)^2 + (700 - 720)^2 \]

\[ \hat{\nu} = \frac{(655 - 670)^2 + (650 - 670)^2 + (625 - 670)^2 + (750 - 670)^2}{2(4-1)} = 3475 \]

\[ \hat{a} = \frac{(720 - 695)^2 + (670 - 695)^2}{2 - 1} - \frac{3475}{4} = 381.25 \]

\[ \hat{k} = 3475/381.25 = 9.1148 \]

\[ \hat{\theta} = \frac{4}{4 + 9.1148} = .305 \]

\[ P_c = .305(670) + .695(695) = 687.4. \]

Question #16
Key: B

There are 430 observations. The expected counts are 430(.2744) = 117.99, 430(.3512) = 151.02, 430(.3744) = 160.99. The test statistic is

\[ \frac{(112 - 117.99)^2}{117.99} + \frac{(180 - 151.02)^2}{151.02} + \frac{(138 - 160.99)^2}{160.99} = 9.15. \]

Question #17
Key: C

See pages 493-98 of the text.
Question #18
Key: B

From the information, the asymptotic variance of $\hat{\theta}$ is $1/4n$. Then
$\text{Var}(2\hat{\theta}) = 4\text{Var}(\hat{\theta}) = 4(1/4n) = 1/n$.
Note that the delta method is not needed for this problem, although using it leads to the same answer.

Question #19
Key: A

The posterior probability density is
$\pi(p \mid 1,1,1,1,1,1,1,1) \propto \text{Pr}(1,1,1,1,1,1,1,1 \mid p)\pi(p) \propto p^8(2) \propto p^8$.

$\pi(p \mid 1,1,1,1,1,1,1,1) = \frac{p^8}{\int_0^5 p^8 dp} = \frac{p^8}{(5^9)/9} = 9(5^{-9})p^8$.

$\text{Pr}(X_o = 1 \mid 1,1,1,1,1,1,1,1) = \int_0^5 \text{Pr}(X_o = 1 \mid p)\pi(p \mid 1,1,1,1,1,1,1,1)dp$

$= \int_0^5 p9(5^{-9})p^8 dp = 9(5^{-9})(5^{10})/10 = .45$.

Question #20
Key: A

The restricted model is $Y = X + \epsilon$, and so $\hat{Y} = X$, and the ESS is $\sum_{i=1}^5 (Y_i - X_i)^2 = 99,374$. The $F$ statistic is $[(99,374 - 69,843)/2]/[69,843/3] = 0.6$, which is less than the 95th percentile of the $F$ distribution with 2 and 3 degrees of freedom.

Question #21
Key: A

$\hat{p}_i = \frac{18}{27} \frac{26}{32} \frac{20}{25} = \frac{13}{30}$
Greenwood’s approximation is

$\left(\frac{13}{30}\right)^2 \left(\frac{9}{18(27)} + \frac{6}{26(32)} + \frac{5}{20(25)}\right) = .0067$. 

Question #22  
Key: D 

\[ \hat{H}(3) = \frac{5}{30} + \frac{9}{27} + \frac{6}{32} = 0.6875 \]

\[ \text{Var}(\hat{H}(3)) = \frac{5}{(30)^2} + \frac{9}{(27)^2} + \frac{6}{(32)^2} = 0.02376 \]

The 95% log-transformed confidence interval is:

\[ \hat{H}(3) U, \text{ where } U = \exp \left( \pm 1.96 \sqrt{\frac{0.02376}{0.6875}} \right) = \exp(\pm 0.43945) \]

The confidence interval is:

\[ [0.6875 \exp(-0.43945), 0.6875 \exp(0.43945)] = [0.443, 1.067]. \]

Question #23  
Key: D 

The means are \(.5(250) + .3(2,500) + .2(60,000) = 12,875\) and \(.7(250) + .2(2,500) + .1(60,000) = 6,675\) for risks 1 and 2 respectively.

The variances are \(.5(250)^2 + .3(2,500)^2 + .2(60,000)^2 - 12,875^2 = 556,140,625\) and \(.7(250)^2 + .2(2,500)^2 + .1(60,000)^2 - 6,675^2 = 316,738,125\) respectively.

The overall mean is \((2/3)(12,875) + (1/3)(6,675) = 10,808.33\) and so

\[ \text{EPV} = (2/3)(556,140,625) + (1/3)(316,738,125) = 476,339,792\] and

\[ \text{VHM} = (2/3)(12,875)^2 + (1/3)(6,675)^2 - 10,808.33^2 = 8,542,222. \] Then,

\[ k = \frac{476,339,792}{8,542,222} = 55.763 \text{ and } Z = 1/(1 + 55.763) = .017617. \]

The credibility estimate is \(.017617(250) + .982383(10,808.33) = 10,622. \]

Question #24  
Key: D 

The first two sample moments are 15 and 500, and the first two population moments are

\[ E(X) = .5(\theta + \sigma) \text{ and } E(X^2) = .5(2\theta^2 + 2\sigma^2) = \theta^2 + \sigma^2. \] These can be obtained either through integration or by recognizing the density function as a two-point mixture of exponential densities. The equations to solve are \(30 = \theta + \sigma\) and \(500 = \theta^2 + \sigma^2\).

From the first equation, \(\sigma = 30 - \theta\) and substituting into the second equation gives

\[ 500 = \theta^2 + (30 - \theta)^2 = 2\theta^2 - 60\theta + 900. \] The quadratic equation has two solutions, 10 and 20. Because \(\theta > \sigma\) the answer is 20.
Question #25  
Key: E

To see that (E) is false, take expectations on both sides of the model:
\[ E(y_1) = .8E(y_{-1}) + 2 + E(\varepsilon_1) - .5E(\varepsilon_{-1}) \]
\[ \mu = .8\mu + 2 + 0 - 0 \]
\[ \mu = 10 \]
The other answers can be shown to be true be looking at various properties of an ARMA(1,1) model.

Question #26  
Key: D

There are four possible samples, (5,5), (5,9), (9,5), and (9,9). For each, the estimator \( g \) must be calculated. The values are 0, 4, 4, and 0 respectively. Assuming a population in which the values 5 and 9 each occur with probability .5, the population variance is \( .5(5 - 7)^2 + .5(9 - 7)^2 = 4 \). The mean square error is approximated as \( .25[(0 - 4)^2 + (4 - 4)^2 + (4 - 4)^2 + (0 - 4)^2] = 8 \).

Question #27  
Key: B

From the Poisson distribution, \( \mu(\lambda) = \lambda \) and \( v(\lambda) = \lambda \). Then,
\[ \mu = E(\lambda) = 6/100 = .06, \quad EPV = E(\lambda) = .06, \quad VHM = Var(\lambda) = 6/100^2 = .0006 \]
where the various moments are evaluated from the gamma distribution. Then,
\[ k = .06/.0006 = 100 \]
\[ Z = 450/(450 + 100) = 9/11 \]
where the 450 is the total number of insureds contributing experience. The credibility estimate of the expected number of claims for one insured in month 4 is \( (9/11)(25/450) + (2/11)(.06) = .056364 \). For 300 insureds the expected number of claims is \( 300(.056364) = 16.9 \).

Question #28  
Key: C

The likelihood function is \( L(\alpha, \theta) = \prod_{j=1}^{200} \frac{\alpha^{\theta^{x_j}}}{(x_j + \theta)^{\alpha+1}} \) and its logarithm is
\[ l(\alpha, \theta) = 200 \ln(\alpha) + 200\alpha \ln(\theta) - (\alpha + 1) \sum_{i=1}^{200} \ln(x_i + \theta) \]. When evaluated at the hypothesized values of 1.5 and 7.8, the loglikelihood is \(-821.77\). The test statistic is \( 2(821.77 - 817.92) = 7.7 \).
With two degrees of freedom (0 free parameters in the null hypothesis versus 2 in the alternative), the test statistic falls between the 97.5\(^{th}\) percentile (7.38) and the 99\(^{th}\) percentile (9.21).
Question #29
Key: B

The least squares estimate is \( \hat{\beta} = \frac{\sum X_i Y_i}{\sum X_i^2} \) and so the first residual and its variance are

\[
\hat{e}_1 = Y_1 - \hat{\beta} X_1 = \beta X_1 + e_1 - \frac{X_1 (\beta X_1 + e_i)}{\sum X_i^2} = \frac{\sum X_i e_i}{X_1^2} = e_1 - \frac{1}{14} (2e_1 + 3e_3)
\]

\[
Var(\hat{e}_1) = \frac{169(1) + 4(9) + 9(16)}{196} = \frac{349}{196} = 1.78.
\]

Question #30
Key: E

Assume that \( \theta > 5 \). Then the expected counts for the three intervals are

\( 15(2/\theta), 15(3/\theta), 15(5/\theta) \) respectively. The quantity to minimize is

\[
\frac{1}{5} \left[ (30\theta^{-1} - 5)^2 + (45\theta^{-1} - 5)^2 + (15 - 75\theta^{-1} - 5)^2 \right].
\]

Differentiating (and ignoring the coefficient of 1/5) gives the equation

\[
-2(30\theta^{-1} - 5)30\theta^{-2} - 2(45\theta^{-1} - 5)45\theta^{-2} + 2(10 - 75\theta^{-1})75\theta^{-2} = 0.
\]

Multiplying through by \( \theta^3 \) and dividing by 2 reduces the equation to

\( -(30 - 5\theta)30 - (45 - 5\theta)45 + (10\theta - 75)75 = -8550 + 1125\theta = 0 \) for a solution of

\( \hat{\theta} = 8550/1125 = 7.6 \).

Question #31
Key: E

\( \pi(\theta | 1) \propto \theta(1.5\theta^5) \propto \theta^{1.5} \). The required constant is the reciprocal of \( \int_0^1 \theta^{1.5} d\theta = \theta^{2.5} \big|_0^1 = .4 \)
and so \( \pi(\theta | 1) = 2.5\theta^{1.5} \). The requested probability is

\[
Pr(\theta > .6 | 1) = \int_6^{1} 2.5\theta^{1.5} d\theta = \theta^{2.5} \big|_6^1 = 1 - .6^{2.5} = .721.
\]
Question #32
Key: A

<table>
<thead>
<tr>
<th>$k$</th>
<th>$kn_k / n_{k-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>1</td>
<td>0.81</td>
</tr>
<tr>
<td>2</td>
<td>0.92</td>
</tr>
<tr>
<td>3</td>
<td>1.75</td>
</tr>
<tr>
<td>4</td>
<td>2.29</td>
</tr>
<tr>
<td>5</td>
<td>2.50</td>
</tr>
<tr>
<td>6</td>
<td>3.00</td>
</tr>
</tbody>
</table>

Positive slope implies that the negative binomial distribution is a good choice. Alternatively, the sample mean and variance are 1.2262 and 1.9131 respectively. With the variance substantially exceeding the mean, the negative binomial model is again supported.

Question #33
Key: B

The required equation is

$$(1 - .7B)(1 - B)(1 + \psi_1B + \cdots) = 1 + .3B$$

$1 + (\psi_1 - 1.7)B + \cdots = 1 + .3B$

$\psi - 1.7 = .3, \quad \psi_1 = 2.$

The variance of the forecast error two steps ahead is $\sigma^2(1 + \psi_1^2) = 1(1 + 2^2) = 5.$

Question #34
Key: B

The likelihood function is

$$e^{-1/(2\theta)} \cdot e^{-2/(2\theta)} \cdot e^{-3/(2\theta)} \cdot e^{-15/(3\theta)} = e^{-8/\theta}.$$  The loglikelihood function is

$$-\ln 24 - 4\ln(\theta) - 8/\theta.$$  Differentiating with respect to $\theta$ and setting the result equal to 0 yields

$$-\frac{4}{\theta} + \frac{8}{\theta^2} = 0$$  which produces $\hat{\theta} = 2.$
Question #35

Key: E

The absolute difference of the credibility estimate from its expected value is to be less than or equal to $k\mu$ (with probability $P$). That is,

$$\left| ZX_{\text{partial}} + (1 - Z)M \right| - \left| Z\mu + (1 - Z)M \right| \leq k\mu$$

$$-k\mu \leq ZX_{\text{partial}} - Z\mu \leq k\mu.$$ 

Adding $\mu$ to all three sides produces answer choice (E).

Question #36

Key: C

$$\sigma^2 = 282.82/(15-4) = 25.71. \quad \text{Var}(\hat{\beta}_2 - \hat{\beta}_2) = \text{Var}(\hat{\beta}_2) + \text{Var}(\hat{\beta}_2) - 2\text{Cov}(\hat{\beta}_2, \hat{\beta}_2)$$

which can be estimated as $25.71[2.14 + 0.03 - 2(0.11)] = 50.14$. The standard error is the square root of the estimated variance, 7.1.

Question #37

Key: C

In general,

$$E(X^2) - E[(X - 150)^2] = \int_0^{200} x^2 f(x) dx - \int_{150}^{200} x^2 f(x) dx - 150^2 \int_{150}^{200} f(x) dx = \int_{150}^{200} (x^2 - 150^2) f(x) dx.$$

Assuming a uniform distribution, the density function over the interval from 100 to 200 is $6/7400$ (the probability of $6/74$ assigned to the interval divided by the width of the interval). The answer is

$$\int_{150}^{200} (x^2 - 150^2) \frac{6}{7400} dx = \left( \frac{x^3}{3} - 150^2 x \right) \frac{6}{7400} \bigg|_{150}^{200} = 337.84.$$

Question #38

Key: C

See pages 476-477.
Question #39
Key: B

The probabilities are from a binomial distribution with 6 trials. Three successes were observed.

\[
\begin{align*}
\Pr(3 \mid I) &= \binom{6}{3} (.1)^3 (.9)^3 = .01458, \\
\Pr(3 \mid II) &= \binom{6}{3} (.2)^3 (.8)^3 = .08192, \\
\Pr(3 \mid III) &= \binom{6}{3} (.4)^3 (.6)^3 = .27648
\end{align*}
\]

The probability of observing three successes is \(.7(.01458) + .2(.08192) + .1(.27648) = .054238.\)

The three posterior probabilities are:

\[
\begin{align*}
\Pr(I \mid 3) &= \frac{.7(.01458)}{.054238} = .18817, \\
\Pr(II \mid 3) &= \frac{.2(.08192)}{.054238} = .30208, \\
\Pr(III \mid 3) &= \frac{.1(.27648)}{.054238} = .50975
\end{align*}
\]

The posterior probability of a claim is then \(.1(.18817) + .2(.30208) + .4(.50975) = .28313.\)

Question #40
Key: E

\[.542 = \hat{F}(n) = 1 - e^{-\hat{H}(n)}, \quad \hat{H}(n) = .78. \quad \text{The Nelson-Aalen estimate is the sum of successive } \frac{s}{r} \text{ values. From the problem statement, } r = 100 \text{ at all surrender times while the } s \text{-values follow the pattern 1, 2, 3, ... Then,}
\]

\[.78 = \frac{1}{100} + \frac{2}{100} + \cdots + \frac{n}{100} = \frac{n(n+1)}{200} \quad \text{and the solution is } n = 12.\]