

COURSE 6
MORNING SESSION

SECTION A – WRITTEN ANSWER

****BEGINNING OF EXAMINATION****
MORNING SESSION

1. (4 points) Outline the key characteristics of securities regulations and restrictions in effect in the United States.

2. (6 points) You are given the following:

| | Scenario 1 | Scenario 2 | Scenario 3 |
|----------------|------------|------------|------------|
| Probability | 45% | 40% | 15% |
| Stock A Return | 30% | 2% | -10% |
| Stock B Return | -8% | 15% | 5% |
| Stock C Return | 8% | 4% | -10% |
| T-bills Return | 3% | 3% | 3% |

An investor has:

- 7,000 invested in Stock A which he cannot sell
- a risk aversion of 4
- 3,000 of additional funds to invest

- (a) Calculate the expected return and standard deviation of each available investment.
- (b) The investor can invest the additional funds in only one investment.
- (i) Calculate the risk and reward of each investment strategy.
 - (ii) Rank each of the investment strategies. Explain your answer.

Show all work.

3. (9 points) You are given the following:

- two types of bonds are available with par values of 100:
 - (i) 5-year zero coupon callable bonds, callable at 80 after two years of call protection
 - (ii) 10-year zero coupon putable bonds, putable to issuer at 40 after three years
 - (iii) market prices are given in the table below:

| Date | Market Price | |
|-------------------|---------------|--------------|
| | Callable Bond | Putable Bond |
| December 31, 2003 | 70 | 50 |
| December 31, 2004 | 100 | 50 |
| December 31, 2005 | 90 | 60 |
| December 31, 2006 | 70 | 60 |

- An investor's strategy is:
 - (i) invest any proceeds received from callable bonds into putable bonds
 - (ii) invest any proceeds received from putable bonds into callable bonds

The investor's initial investment on December 31, 2003 was split between one callable bond and one putable bond. Both bonds purchased had two years of protection remaining at time of purchase.

December 31 of each year is the only day for purchasing, calling or putting bonds. Bonds are called or put whenever the opportunity arises.

Today's date is December 31, 2004.

- (a) Contrast put options with call options.
- (b) Describe the risks associated with the embedded options in the initial investment.
- (c) Calculate the holding period return if all bonds are sold on December 31, 2006. Assume no transaction costs.
- (d) Contrast options with futures.
- (e) Describe how futures could be used to improve the holding period return.

Show all work.

4. (6 points) You are given the following information about three collateralized mortgage obligations (CMOs):

- CMO A is backed by 7.5% pass-throughs consisting of the following tranches:
 - 3-year sequential-pay
 - 5-year very accurately defined maturity (VADM)
 - 7-year sequential-pay
 - 10-year sequential-pay
 - 17-year Z bond

- CMO B is backed by 7.5% pass-throughs consisting of the following tranches:
 - 3-year planned amortization classes (PAC)
 - 7-year PAC
 - 7-year companion
 - 10-year PAC
 - 16-year PAC
 - 20-year companion

- CMO C is backed by 7.5% pass-throughs consisting of the following tranches:
 - 3-year sequential-pay
 - 7-year sequential-pay
 - 10-year sequential-pay
 - 17-year sequential-pay

U.S. federal authorities are expected to increase interest rates by 50 basis points.

Describe how each structure will be affected by the increase.

5. (8 points) You are given the following with respect to two public companies:

- the common shares of each company are currently trading at 30 as of December 31, 2003
- neither company pays shareholder dividends
- there are no taxes or transaction costs
- an industry analyst has projected the possible stock prices over the next two years as a function of the performance of the US economy:

| US Economy | | Company A | | Company B | |
|------------|-----------|-------------------|-------------------|-------------------|-------------------|
| 2004 | 2005 | December 31, 2004 | December 31, 2005 | December 31, 2004 | December 31, 2005 |
| Expansion | Expansion | 32 | 33 | 33 | 36 |
| Expansion | Recession | 32 | 30 | 33 | 30 |
| Recession | Expansion | 30 | 29 | 29 | 29 |
| Recession | Recession | 30 | 30 | 29 | 27 |

- Determine if the analyst's projections allow for arbitrage.
- Using the analyst's projections, determine the value of a European put option on Company B's stock if the option expires on December 31, 2005, and has an exercise price of 30.
- Determine how an investor could replicate the payoff of a one-year European call option with an exercise price of 31 on Company A's stock using a portfolio of the two companies common shares.

Show all work.

6. (5 points) You are given the following:

| Bond | Features | | |
|------|------------------|------------------------|------------------|
| | Maturity (years) | Annual Coupon Rate (%) | Embedded Options |
| A | 10 | 5% | None |
| B | 10 | 5% | Put Option |
| C | 10 | 7% | None |
| D | 10 | 7% | Call Option |
| W | 20 | 8% | None |
| X | 20 | 8% | Put Option |
| Y | 20 | 10% | None |
| Z | 20 | 10% | Call Option |

- (a) Describe how bond features affect interest rate risk.
- (b) An investor only buys bonds that have at least two of their features with high sensitivity to interest rate changes. Identify the four bonds that this investor will buy. Explain your answer.

7. (5 points)

- (a) Describe the advantages of simulation techniques and lattice methods.
- (b) Outline the issues that arise when implementing simulation techniques for a mortgage-backed securities portfolio.
- (c) Describe how lattice methods are used in a simulation model when evaluating a mortgage-backed securities portfolio.

**COURSE 6
MORNING SESSION**

SECTION B – MULTIPLE CHOICE

1. You are given the following bond portfolio:

| S&P Rating | Percentage of Portfolio |
|------------|-------------------------|
| AAA | 10% |
| AA+ | 10% |
| BBB+ | 10% |
| A+ | 25% |
| D | 2% |
| CC | 3% |
| BBB- | 15% |
| BB+ | 5% |
| A | 20% |

Determine the percentage of the portfolio that is investment grade.

- (A) 65%
- (B) 75%
- (C) 90%
- (D) 95%
- (E) 100%

USE THIS PAGE FOR YOUR SCRATCH WORK

2. You are given the following information for a mutual fund:

- net asset value (NAV) at December 31, 2003: 28
- income distribution per share in 2004: 0.4
- assets at December 31, 2004: 620,000
- liabilities at December 31, 2004: 14,600
- shares outstanding: 20,000

No securities were sold throughout the year. There are no capital gain distributions and no fees in the year.

Calculate the effective annual interest rate of return for the mutual fund.

- (A) 6.7%
- (B) 8.1%
- (C) 8.8%
- (D) 9.5%
- (E) 12.1%

USE THIS PAGE FOR YOUR SCRATCH WORK

3. A one-period securities market model is given by $S(0) = [1 \ 1 \ 1]$.

$$S(1) = \begin{pmatrix} 2 & 1.6 & 0 \\ 0 & 0.8 & x \\ 0 & 0 & 1 \end{pmatrix}$$

Determine the range of values for x so that this model is arbitrage free.

- (A) $x > 4$
- (B) $x < 4$
- (C) $x > 0$
- (D) $x < 0$
- (E) no such x exists

USE THIS PAGE FOR YOUR SCRATCH WORK

4. You are given the following:

- expected return of market portfolio: 7.0%
- variance of market portfolio: 10.0%
- variance of Security A: 19.0%
- covariance of Security A and market portfolio: 25.0%
- risk-free rate: 5.0%

Calculate the expected return of Security A using CAPM.

- (A) 7.6%
- (B) 10.0%
- (C) 14.2%
- (D) 17.5%
- (E) 22.5%

USE THIS PAGE FOR YOUR SCRATCH WORK

5-9. Questions 5 through 9 consist of an assertion in the left-hand column and a reason in the right-hand column. Code your answer to each question by blackening space:

- (A) If both the assertion and the reason are true statements, and the reason is a correct explanation of the assertion.
- (B) If both the assertion and the reason are true statements, but the reason is NOT a correct explanation of the assertion.
- (C) If the assertion is a true statement, but the reason is a false statement.
- (D) If the assertion is a false statement, but the reason is a true statement.
- (E) If both the assertion and the reason are false statements.

ASSERTION

REASON

5. The original loan-to-value ratio overestimates the measurement of defaults for home mortgages.

BECAUSE

As home mortgages become more seasoned, default rates tend to decline.

ASSERTION

REASON

6. Preferred stock payments are tax-deductible for the issuing corporation.

BECAUSE

Preferred stock payments are treated as dividends.

ASSERTION

7. An option-adjusted spread (OAS) value by itself does not provide sufficient information to determine whether a bond is rich or cheap.

REASON

BECAUSE OAS is the spread to short-term interest rates that equates the theoretical price of a bond to its market price.

ASSERTION

8. Interest rate caps and floors should not be used when the holding period of the underlying asset or liability is flexible or subject to change.

REASON

BECAUSE Termination of interest rate caps and floors involves exit costs.

ASSERTION

9. A trustee may engage in a transaction that breaches another duty to the detriment of the participants because he is instructed to do so in accordance with the trust agreement.

REASON

BECAUSE The trustee is under the obligation to manage the trust in accordance with the trust agreement.

10-16. Each of questions 10 through 16 consists of two lists. In the list at the left are two items, lettered X and Y. In the list at the right are three items, numbered I, II, and III. ONE of the lettered items is related in some way to EXACTLY TWO of the numbered items. Indicate the related items using the following answer code:

| | <u>Lettered Item</u> | <u>Is Related to Numbered Items</u> |
|-----|--|-------------------------------------|
| (A) | X | I and II only |
| (B) | X | II and III only |
| (C) | Y | I and II only |
| (D) | Y | I and III only |
| (E) | The correct answer is not given by (A), (B), (C) or (D). | |

- 10.**
- | | |
|--|--|
| <p>X. Real Assets</p> <p>Y. Financial Assets</p> | <p>I. Patents</p> <p>II. Appear only on the asset side of the balance sheet</p> <p>III. Are created and destroyed in the ordinary course of doing business</p> |
|--|--|

- 11.**
- | | |
|---|---|
| <p>X. Callable Bonds</p> <p>Y. Puttable Bonds</p> | <p>I. Bond price at low yields is approximately equal to the price of an option-free bond.</p> <p>II. Negative convexity at low yield levels and positive convexity at high yield levels.</p> <p>III. Positive convexity at all yield levels.</p> |
|---|---|

- 12.** X. Lattice methods
- I. Allow for more realistic jumps in interest rates of varying amounts along a single path.
- Y. Simulation methods
- II. Easier to use to value instruments that have embedded American-style options.
- III. Easier to use when more than one factor changes at the same time.

- 13.** X. Forward contracts
- I. Dealer or broker
- Y. Futures contracts
- II. Bid-ask spread
- III. High liquidity

- 14.** X. Agency CMOs
- I. Usually rated by bond rating agencies.
- Y. Whole-loan CMOs
- II. Do not employ credit enhancement techniques.
- III. Employ credit enhancement techniques.

10-16. Each of questions 10 through 16 consists of two lists. In the list at the left are two items, lettered X and Y. In the list at the right are three items, numbered I, II, and III. ONE of the lettered items is related in some way to EXACTLY TWO of the numbered items. Indicate the related items using the following answer code:

| | <u>Lettered Item</u> | <u>Is Related to Numbered Items</u> |
|-----|--|-------------------------------------|
| (A) | X | I and II only |
| (B) | X | II and III only |
| (C) | Y | I and II only |
| (D) | Y | I and III only |
| (E) | The correct answer is not given by (A), (B), (C) or (D). | |

- 15.**
- | | | | |
|----|-------------------------|------|---|
| X. | Horizon matching | I. | Active management. |
| Y. | Contingent immunization | II. | Reduces impact of non-parallel move of yield curve. |
| | | III. | Always duration matched. |

- 16.**
- | | | | |
|----|---------------------|------|--|
| X. | Value-at-risk model | I. | Measures the maximum loss in value a company's portfolio is likely to sustain over a period of time as a result of changes in market prices. |
| Y. | Stress test | II. | Measures a company's exposure to extreme movements in stock prices. |
| | | III. | Uses average historical correlations among asset prices. |

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17. You are given the following:

| | |
|---------------------------|---------|
| • issues traded: | 4,800 |
| • advances: | 2,600 |
| • declines | 1,500 |
| • unchanged: | 650 |
| • new highs: | 150 |
| • new lows: | 50 |
| • advancing volume (000): | 470,000 |
| • declining volume (000): | 235,000 |
| • total volume (000): | 740,000 |

Calculate the “trin” statistic.

- (A) 0.87
- (B) 1.02
- (C) 1.15
- (D) 1.17
- (E) 2.00

USE THIS PAGE FOR YOUR SCRATCH WORK

18. An investor purchases 50 shares of a stock on January 28, 2004.

You are given the following:

- annual interest on borrowed funds: 7%
- value of each share on January 28, 2005: 100
- investor's return from January 28, 2004 to January 28, 2005: 30%

Calculate the original purchase price of each share, if the initial margin percentage is 75%.

- (A) 66.23
- (B) 80.48
- (C) 81.63
- (D) 85.06
- (E) 86.02

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19. You are given the following:

- target return on capital: 15%
- net spread for residential mortgages: 1%
- net spread for mortgage bonds: 0.9%

- pre-tax return: 21%
- risk-free rate: 6%

- MCCR factors:
 - BBB mortgage bonds: 2%
 - BB residential mortgages: 4%

Calculate the MCCR-adjusted spread for residential mortgages and mortgage bonds, respectively.

- (A) (0.40%, 0.30%)
- (B) (0.40%, 0.60%)
- (C) (0.64%, 0.72%)
- (D) (0.70%, 0.60%)
- (E) (0.76%, 0.78%)

USE THIS PAGE FOR YOUR SCRATCH WORK

20. You are given the following:

| Risk | Amount |
|------|--------|
| C-1 | 80 |
| C-2 | 100 |
| C-3 | 90 |
| C-4 | 10 |

Using Zeppatella's formula for incremental RBC, determine (W_a, W_i) , where W_a is the weight associated with assets and W_i is the weight associated with insurance.

- (A) (0.743, 0.437)
- (B) (0.793, 0.467)
- (C) (0.793, 0.507)
- (D) (0.862, 0.507)
- (E) (0.862, 0.682)

USE THIS PAGE FOR YOUR SCRATCH WORK

21-24. Questions 21 through 24 consist of an assertion in the left-hand column and a reason in the right-hand column. Code your answer to each question by blackening space:

- (A) If both the assertion and the reason are true statements, and the reason is a correct explanation of the assertion.
- (B) If both the assertion and the reason are true statements, but the reason is NOT a correct explanation of the assertion.
- (C) If the assertion is a true statement, but the reason is a false statement.
- (D) If the assertion is a false statement, but the reason is a true statement.
- (E) If both the assertion and the reason are false statements.

| <u>ASSERTION</u> | | <u>REASON</u> |
|---|---------|--|
| 21. Given the choice, investors prefer a portfolio on a higher indifference curve. | BECAUSE | Higher indifference curves correspond to higher levels of utility. |

| <u>ASSERTION</u> | | <u>REASON</u> |
|--|---------|---|
| 22. A callable bond is negatively convex. | BECAUSE | A call option increases a bond's effective duration when interest rates fall. |

ASSERTION

- 23.** Modified duration is an inappropriate measure for bonds with embedded options.

REASON

BECAUSE Modified duration assumes a flat yield curve.

ASSERTION

- 24.** Dividend scale guarantees are put options within a participating life insurance policy.

REASON

BECAUSE A put option gives the purchaser the right to sell at a fixed price.

25-31. Each of questions 25 through 31 consists of two lists. In the list at the left are two items, lettered X and Y. In the list at the right are three items, numbered I, II, and III. ONE of the lettered items is related in some way to EXACTLY TWO of the numbered items. Indicate the related items using the following answer code:

| | <u>Lettered Item</u> | <u>Is Related to Numbered Items</u> |
|-----|--|-------------------------------------|
| (A) | X | I and II only |
| (B) | X | II and III only |
| (C) | Y | I and II only |
| (D) | Y | I and III only |
| (E) | The correct answer is not given by (A), (B), (C) or (D). | |

- 25.**
- | | | | |
|----|---------------------|------|--|
| X. | Refunding Provision | I. | Will not provide protection against the issuance of common stock to retire debt. |
| Y. | Call Provision | II. | Bonds cannot be redeemed for any reason. |
| | | III. | Provides redemption protection against the issuance of debt ranking equal to or superior to the debt to be redeemed. |

- 26.**
- | | | | |
|----|---------------------------------------|------|---|
| X. | Multivariate density estimation (MDE) | I. | Chooses points which are as uniformly distributed as possible. |
| Y. | Low-discrepancy method | II. | Can only be used for classes of financial instruments for which liquid markets exist. |
| | | III. | Nonparametric, model-free approach. |

- 27.** X. Amortized cost method I. Assets held to maturity
Y. Market value method II. Assets available for sale
III. Assets for trading
- 28.** X. Return simulation I. Evaluates the current level of yields by producing spot, discount, and forward rate structures.
Y. Term-structure analysis II. Predicts bond and portfolio behavior given alternative interest rate scenario projections.
III. Values Treasury securities.
- 29.** X. Duration bets I. Pure index match
Y. No duration bets II. Minor mismatches
III. Full-blown active

25-31. Each of questions 30 through 31 consists of two lists. In the list at the left are two items, lettered X and Y. In the list at the right are three items, numbered I, II, and III. ONE of the lettered items is related in some way to EXACTLY TWO of the numbered items. Indicate the related items using the following answer code:

| | <u>Lettered Item</u> | <u>Is Related to Numbered Items</u> |
|-----|--|-------------------------------------|
| (A) | X | I and II only |
| (B) | X | II and III only |
| (C) | Y | I and II only |
| (D) | Y | I and III only |
| (E) | The correct answer is not given by (A), (B), (C) or (D). | |

- 30.**
- | | | | |
|----|------------------------------|------|-------------------------|
| X. | Price sensitivity techniques | I. | Maturity gap management |
| Y. | Cash flow techniques | II. | Key rate duration |
| | | III. | Convexity |

- 31.**
- | | | | |
|----|----------------------------|------|---|
| X. | Z Bonds | I. | No reinvestment risk during the accretion phase. |
| Y. | Accretion-directed Classes | II. | Do not extend even if there are no prepayments. |
| | | III. | Price is highly sensitive to interest rate movements. |

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32. You are given the following information with respect to a portfolio:

- duration: 3
- dispersion: 0.75
- all cash flows are positive

| Time (t) | $PV(CF_t)$ |
|--------------|------------|
| 1 | 1 |
| 2 | X |
| 3 | Y |
| 4 | 0 |
| 5 | 1 |

Determine the minimum value of Y .

- (A) 0
- (B) 3
- (C) 4
- (D) 9
- (E) 26

USE THIS PAGE FOR YOUR SCRATCH WORK

- 33.** You are given the following with respect to a multiplicative binomial model:
- the current short rate is 4%
 - rates are twice as likely to rise as they are to fall
 - the volatility parameter is 20%

Determine the expected short rate two periods from now.

- (A) 3.65
- (B) 4.13
- (C) 4.31
- (D) 4.55
- (E) 4.65

USE THIS PAGE FOR YOUR SCRATCH WORK

34. You are given the following with respect to a mutual fund:

- annual expense ratio: 1%
- gross annual rate of return: 10%

The back-end load fee starts at 4% and reduces by 0.5% on each anniversary.
An investor makes an initial investment of 5,000 on January 1, 2002.

Calculate the realized gain if the investor sells the shares on December 31, 2004.

- (A) 941
- (B) 1,216
- (C) 1,281
- (D) 1,313
- (E) 1,455

**COURSE 6
AFTERNOON SESSION**

SECTION C – WRITTEN ANSWER

****BEGINNING OF EXAMINATION****
AFTERNOON SESSION

8. (4 points) Describe the practical difficulties that may be encountered while implementing an asset allocation optimization model.

9. (5 points) You are given the following information about a universe of securities:

- R_f : 0.0300
- $E(R_M)$: 0.0700
- S^2_M : 0.0450

The market covariance grid is:

| Security | Market Weight | Covariance Matrix | | | |
|----------|---------------|-------------------|-----|----|-----|
| A | 0.35 | 9% | 2% | 2% | 8% |
| B | 0.25 | 2% | 12% | 1% | 2% |
| C | 0.25 | 2% | 1% | 3% | 5% |
| D | 0.15 | 8% | 2% | 5% | 12% |

- (a) Calculate the market price of risk.
- (b) Calculate the contribution to the variance of the market portfolio for securities B and D.
- (c) Calculate the equilibrium expected rate of return for securities B and D.

Show all work.

- 10.** (6 points) You are given the following with respect to T-bills issued on February 1, 2004:

| Term to Maturity | Spot Price |
|------------------|------------|
| 1 month | 98.50 |
| 2 months | 98.40 |
| 3 months | 98.20 |
| 4 months | 98.00 |

A futures contract is available for delivery of a 3-month T-bill on March 1, 2004.

- (a) Compare forward contracts to futures contracts.
- (b) Determine the implied price of the futures contract.
- (c) Recommend an arbitrage strategy to be implemented on February 1, 2004 if the market price of the futures contract at that time is 97.5. Calculate the profit and cash flows each month until all contracts are mature.

Show all work.

- 11.** (6 points) You are given the following with respect to a one-period interest rate contingent claim:

- martingale probability: 50%
- variance of the security: 0.2755%
- risk-free rate: 5%
- payment if interest rates go up: 105
- payment if interest rates go down: 95

- (a) Calculate the martingale security price.
- (b) Calculate the “true” market price of risk.
- (c) Calculate the “true” probability.

Show all work.

12. (8 points) You are given the following:

| Time to Maturity | Effective Annual Spot Rate |
|------------------|----------------------------|
| 0.5 years | 5.00% |
| 1.5 years | 5.50% |
| 2.5 years | 6.10% |
| 3.5 years | 6.25% |
| 4.5 years | 6.50% |

- the valuation date is December 31, 2004
- liability cash flows are as follows:

| Time of Cash Flow | Liability Amount |
|-------------------|------------------|
| June 30, 2005 | 25 |
| June 30, 2006 | 20 |
| June 30, 2007 | 15 |
| June 30, 2008 | 10 |
| June 30, 2009 | 5 |

- the Macauley duration of the liabilities is 1.75 years
- the convexity of the liabilities is 5.5 years
- the universe of publicly traded bonds at 100 par value as at December 31, 2004 is as follows:

| Bond | Maturity | Annual Coupon | Effective Annual Yield |
|------|---------------|---------------|------------------------|
| A | June 30, 2005 | 10.0% | 5.00% |
| B | June 30, 2006 | 7.5% | 5.50% |
| C | June 30, 2007 | 6.5% | 6.50% |
| D | June 30, 2008 | 5.0% | 6.50% |
| E | June 30, 2009 | 4.0% | 6.50% |

- Calculate the Macauley duration of Bond B.
- Compare immunization and cash flow matching strategies.
- Construct an immunized portfolio using Bond A and Bond E that protects against interest rate movements.
- Determine a cash flow matched portfolio using the universe of available bonds.

Show all work.

- 13.** (6 points) A bond portfolio consists of three bonds, each issued at the same par value. You are given the following projected results over a one-year planning horizon under three different interest rate scenarios.

| Scenario | Bond | Total Annual Return % | Duration |
|----------|------|-----------------------|----------|
| 1 | X | 11.9 | 2.3 |
| | Y | 10.1 | 1.4 |
| | Z | 10.7 | 1.1 |
| 2 | X | 8.2 | 2.3 |
| | Y | 8.4 | 1.4 |
| | Z | 8.3 | 1.1 |
| 3 | X | 5.7 | 2.3 |
| | Y | 6.1 | 1.4 |
| | Z | 5.6 | 1.1 |

Each of the three scenarios is equally likely to occur and the total portfolio consists of one of each of the bonds.

- Describe Strategic Frontier Analysis.
- Using the Strategic Frontier Analysis, plot the graph and evaluate each of the bonds.
- Describe Relative Return Value Analysis.
- Using the Relative Return Value Analysis, plot the graph and evaluate each of the bonds.

Show all work.

14. (5 points) You are given the following with respect to a Treasury instrument:

- days to maturity: 110
- bid: 4.48%
- ask: 4.46%
- change: +0.02%
- ask yield: 4.50%

- (a) Calculate as of today:
- (i) Market Price
 - (ii) Bond Equivalent Yield
 - (iii) Effective Annual Yield
- (b) Describe the characteristics of U.S. Treasuries and how they are made available to markets.
- (c) Describe the characteristics of Agency Securities and how they are made available to markets.

Show all work.

COURSE 6
Spring 2004

Multiple-Choice Answer Key

| <i>Question #</i> | <i>Answer</i> | | <i>Question #</i> | <i>Answer</i> |
|-------------------|----------------------------|--|-------------------|---------------|
| 1 | C | | 21 | A |
| 2 | D | | 22 | C |
| 3 | B | | 23 | A |
| 4 | B | | 24 | D |
| 5 | D | | 25 | E |
| 6 | D | | 26 | B |
| 7 | B | | 27 | E |
| 8 | A | | 28 | D |
| 9 | D | | 29 | C |
| 10 | Correct for All | | 30 | B |
| 11 | D | | 31 | E |
| 12 | D | | 32 | D |
| 13 | A | | 33 | E |
| 14 | D | | 34 | C |
| 15 | B | | | |
| 16 | E | | | |
| 17 | A | | | |
| 18 | B | | | |
| 19 | B | | | |
| 20 | D | | | |

Spring 2004 Course 6 Written-Answer Solutions

Question #1 Solution

- Securities Exchange Act of 1933
Required registration of prospectus with SEC
Required full disclosure of all relevant information relating to the issuance of new securities
This does not mean that the security is a “good buy” but only that relevant information is provided to investors
- Securities Exchange Act of 1934
Established the Securities & Exchange Commission (SEC) in order to administer the 1933 Act
Gave the SEC the authority to register and regulate securities exchanges, over-the-counter (OTC) markets, brokers, and dealers
- Securities Investment Protection Corporation (SIPC)
Established in 1970 in order to order protect investors from losses that occur when their brokerage firms fail
- State laws
Securities trading is also subject to state laws, known as “blue sky” laws, which attempt to prevent false sale and promotion of securities
- Insider Trading
It is strictly illegal for anyone to profit by trading based on inside information; that is, information that has not yet been released to the public
- Circuit Breakers
These help regulate the securities exchanges directly
There are two basic types
 - Trading Halts
If DJIA falls 10%, trading is halted for one hour
If DJIA falls 20%, trading is halted for two hours
If DJIA falls 30%, trading stops for the day
 - Collars
If DJIA moves either up or down by more than 210 points within the course of a day’s trading, all trades are required to pass a “tick test.” That is, the trade will only be executed if the last price movement was positive.

Question #2 Solution

(a) Expected return = $\sum r_i p(s)$

Standard deviation = $\sqrt{\sum (r_i - E(r_i))^2 p(s)}$

For Stock A

Expected return = $[30\% \times 45\%] + [2\% \times 60\%] + [(-10\%) \times 15\%] = 12.8\%$

Variance =

$[(30\% - 12.8\%)^2 \times 45\%] + [(2\% - 12.8\%)^2 \times 40\%] + [(-10\% - 12.8\%)^2 \times 15\%] = 257.76$

Standard Deviation = $\sqrt{257.76} \doteq 16.05\%$

For Stock B

Expected return = $[8\% \times 45\%] + [15\% \times 40\%] + [5\% \times 15\%] = 3.15\%$

Variance =

$[(-8 - 3.15)^2 \times 45\%] + [(15 - 3.15)^2 \times 40\%] + [(5 - 3.15)^2 \times 15\%] = 112.6275$

Standard deviation = $\sqrt{112.6275} \doteq 10.61\%$

For Stock C

Expected return = $[8\% \times 45\%] + [4\% \times 40\%] + [(-10\%) \times 15\%] = 3.7\%$

Variance = $[(8 - 3.7)^2 \times 45\%] + [(6 - 3.7)^2 \times 40\%] + [(-10 - 3.7)^2 \times 15\%] = 36.51$

Standard deviation = $\sqrt{36.51} = 6.04\%$

For T-bill

Expected return = 3%

Standard deviation = 0%

- (b) (i) Condition 1 = invest 3000 in Stock A \Rightarrow 100% invest in Stock A
Expected return = 12.8%
Standard deviation = 16.05%

$$r_p = w_A r_A + w_B r_B$$

$$\Rightarrow E(r_p) = w_A E(r_A) + w_B E(r_B)$$

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \text{cov}(r_A, r_B)$$

Condition 2 – invest 3000 in Stock B

$$\begin{aligned} & 70\% \text{ in Stock A} \\ \Rightarrow & 30\% \text{ in Stock B} \\ & r_p = 0.7r_A + 0.3r_B \end{aligned}$$

$$E(r_p) = [0.7 \times 12.8\%] + [0.3 \times 3.15\%] = 9.905\%$$

$$Cov(r_A, r_B) = E_p(s)(r_A - E(r_A))(r_B - E(r_B))$$

$$= 0.45 \times (30\% - 12.8\%) \times (-8 - 3.15\%)$$

$$+ 0.4 \times (2\% - 12.8\%) \times (15 - 3.15\%)$$

$$+ 0.15 \times (-10\% - 12.8\%) \times (5 - 3.15\%) = 143.82$$

$$\sigma_p^2 = [0.7^2 \times 16.05^2] + [0.3^2 \times 10.61^2] + [2 \times 0.7 \times 0.3 \times (-143.82)] = 75.952314$$

$$\sigma_p \doteq 8.72\%$$

Condition 3 – invest 3000 in Stock C

$$\begin{aligned} & 70\% \text{ in Stock A} \\ \Rightarrow & 30\% \text{ in Stock C} \\ & r_p = 0.7r_A + 0.3r_C \end{aligned}$$

$$E(r_p) = 0.7 \times 12.8\% + 0.3 \times 3.7\% = 10.07\%$$

$$Cov(r_A, r_C) = 0.45 \times (30 - 12.8)(8 - 3.7)$$

$$+ 0.4 \times (2 - 12.8)(4 - 3.7)$$

$$+ 0.15 \times (-10 - 12.8)(-10 - 3.7)$$

$$= 78.84$$

$$\sigma_p^2 = [0.7^2 \times 16.05^2] + [0.3^2 \times 6.04^2] + [2 \times 0.7 \times 0.3 \times 78.84]$$

$$= 162.621369$$

$$\sigma_p \doteq 12.75\%$$

Condition 4 – invest 3000 in T-bill

$$\begin{aligned} & 70\% \text{ in Stock A} \\ \Rightarrow & 30\% \text{ in T-bill} \\ & r_p = 0.7r_A + 0.3r_{T\text{-bill}} \end{aligned}$$

$$E(r_p) = [0.7 \times 12.8\%] + [0.3 \times 3\%] = 9.86\%$$

$$\sigma_p^2 = 0.7^2 \times 16.05^2 = 126.225225$$

$$\sigma_p = 11.235\%$$

$$(ii) \quad A = \text{risk aversion} = 4 \quad U = E(r_p) - 0.005A\sigma_p^2$$

Condition 1 = All in Stock A

$$U = 12.8 - 0.005 \times 4 \times 16.05^2 = 7.64795$$

Condition 2 = 70% in Stock A, 30% in Stock B

$$U = 9.905 - 0.005 \times 4 \times 75.952314 = 8.38595372$$

Condition 3 = 70% in Stock A, 30% in Stock C

$$U = 10.07 - 0.005 \times 4 \times 162.621369 = 6.81757262$$

Condition 4 = 70% in Stock A, 30% in T-bill

$$U = 9.86 - 0.005 \times 4 \times 126.225225 = 7.3354955$$

Rank by utility: (high \rightarrow low)

Condition 2 = 3000 invest in Stock B

Condition 1 = 3000 invest in Stock A

Condition 4 = 3000 invest in T-bill

Condition 3 = 3000 invest in Stock C

- (d) option
- right to buy (call) or sell (put) a bond at preset price and date (maturity if European, earlier if American)
 - up front premium paid for this right
- futures
- obligation to buy or sell a bond at present price and date
 - no upfront premium is required
 - back by the exchange
 - standardized
- (e) Futures could be used to improve holding period return by:
- hedge against price changes that would cause the bond to be called early
 - lock in a set return
 - enter into a futures contract that sells the bond @12/31/06 at a higher price (each party is betting that interest rates will move in opposite direction
($i \uparrow P \downarrow$) = sells short position, ($i \downarrow P \uparrow$) = long position)

Question #4 Solution

CMO A

- an increase in interest rates will reduce prepayments
- extension risk will become an issue
- the Z bonds principal and interest payment are used to accelerate principal repayments to the sequential pay classes and reduce their volatility
- the Z bond will only receive payments once the sequential classes have been paid off
- the VADM is protected against extension risk, (as the payments to the VAAM are from the interest accretion of the Z bond, the VADM will follow its schedule until the Z bond begins to paydown)
- the sequential classes will paydown as normal, from the shortest tranche to the longest

CMO B

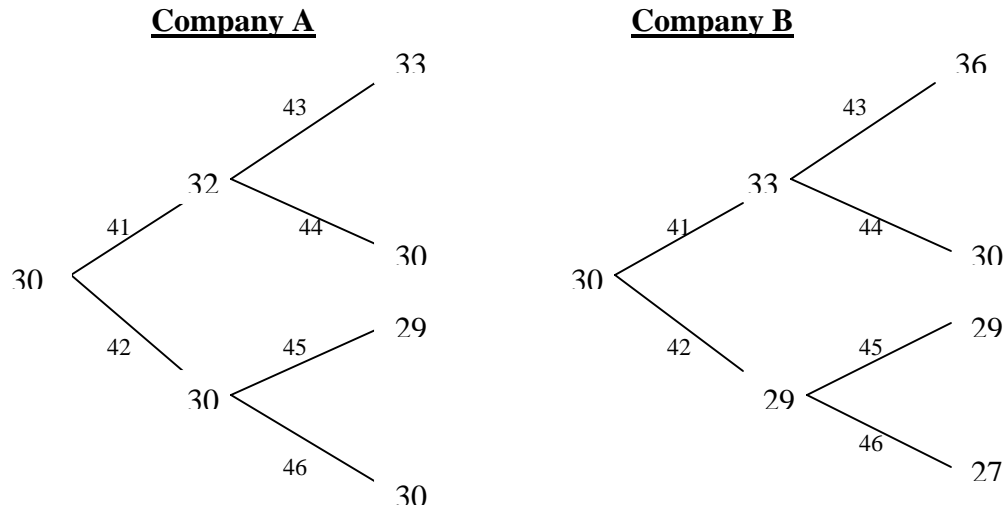
- PAC's have prepayment protection from their companion classes
- The companion classes will absorb any prepayment fluctuations
- As long as the prepayments are within the PAC ranges, the principals received will be stable
- As the prepayment will slow down, the prepayment bands for the PAC's will move
- if the prepayments are below the lower bands
 - => the upper and lower bands will rise
 - => the lower band rises because of built in catch up features
 - => the upper band rises because the companions can accept more prepayment fluctuations
- if prepayments are above the lower band but below the upper bands
 - => both bands will move up
 - => the lower band because the remaining outstanding principal is less so the prepayments have to increase to maintain previous prepayment levels

CMO C

- the principal payments will slow down due to prepayments
- for sequential classes the principal is paid to the shortest tranche first (in this case the year tranche); once that tranche is paid off the next shortest tranche will receive principals
- due to lower prepayments, the tranches will receive their principal payments later
- interest payments are to all tranches, based on outstanding principal (will be higher as payment of principal is slower)

Question #5 Solution

(a)



$$32\varphi_1 + 30\varphi_2 = 30 \rightarrow 32\varphi_1 = 30 - 30\varphi_2 \rightarrow \varphi_1 = 0.9375 - 0.9375\varphi_2$$

$$33\varphi_1 + 29\varphi_2 = 30$$

$$33(0.9375 - 0.9375\varphi_2) + 29\varphi_2 = 30$$

$$30.9375 - 30.9375\varphi_2 + 29\varphi_2 = 30 \quad \varphi_2 = 0.4839 \quad \varphi_1 = 0.4838$$

$$33\varphi_3 + 30\varphi_4 = 32 \rightarrow 33\varphi_3 = 32 - 30\varphi_4 \rightarrow \varphi_3 = 0.9697 - 0.9091\varphi_4$$

$$36\varphi_3 + 30\varphi_4 = 33$$

$$36(0.9697 - 0.9091\varphi_4) + 30\varphi_4 = 33$$

$$34.9092 - 32.7276\varphi_4 + 30\varphi_4 = 33 \quad \varphi_4 = 0.70 \quad \varphi_3 = 0.3333$$

$$29\varphi_5 + 30\varphi_6 = 30 \rightarrow 29\varphi_5 = 30 - 30\varphi_6 \rightarrow \varphi_5 = 1.0345 - 1.0345\varphi_6$$

$$29\varphi_5 + 27\varphi_6 = 29$$

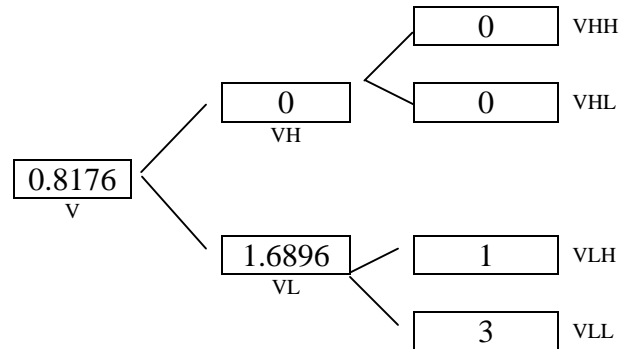
$$29(1.0345 - 1.0345\varphi_6) + 27\varphi_6 = 29$$

$$30 - 30\varphi_6 + 27\varphi_6 = 29$$

$$-3\varphi_6 = -1 \quad \varphi_6 = \frac{1}{3} \quad \varphi_5 = 0.6897$$

Because there is a state price vector, no arbitrage exists.

- (b) European Put option → option to sell stock at 30 at maturity date; will exercise if price less than 30



$$v_{LH} = \text{Max}(O, K - S) = \text{Max}(0, 30 - 29) = 1$$

$$v_{LL} = \text{Max}(O, K - S) = \text{Max}(0, 30 - 27) = 3$$

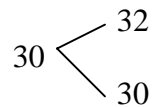
$$v_H = 0$$

$$v_L = 1\phi_5 + 3\phi_6 = 1(0.6897) + 3(0.3333) = 1.6896$$

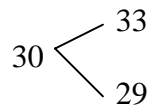
$$v = 0\phi_1 + 1.6896\phi_2 = 0 + 1.6896(0.48939) = 0.8176$$

- (c)

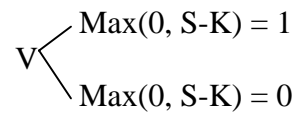
Company A



Company B



Call Option on A



$$32\theta_1 + 33\theta_2 = 1 \rightarrow 32\theta_1 = 1 - 33\theta_2 \rightarrow \theta_1 = \frac{1 - 33\theta_2}{32}$$

$$30\theta_1 + 29\theta_2 = 0$$

$$30\left(\frac{1 - 33\theta_2}{32}\right) + 29\theta_2 = 0$$

$$0.9375 - 30.9375\theta_2 + 29\theta_2 = 0$$

$$0.9375 = 1.9375\theta_2 \quad \theta_2 = 0.4839$$

$$\theta_1 = -0.4678$$

Can replicate call option by purchasing 0.4839 of Company A stock and selling 0.4678 of Company B stock.

$$\text{Cost} = 30(0.4839) + 30(-0.4678) = 0.483$$

Question #6 Solution

(a) Impact of Maturity

- all other factors constant, the longer the bond's maturity, the greater the bond's sensitivity to interest rate changes

Impact of Coupon Rate

- all other factors constant, the lower the coupon rate, the greater the bond's sensitivity to changes in interest rates
- zero coupon bonds have a greater price sensitivity to interest rate changes than same-maturity bonds bearing a coupon rate and trading at the same yield

Impact of Embedded Options

- Price of Callable bond = Price of Option-free Bond – Price of Call Option
- when interest rates decline, the price of the Option-free Bond will increase and the price of the Call Option will also increase
- the price of the Callable bond may increase or decrease depending on the relative changes in the Option-free Bond price and the Call Option price
- typically, the price of a Callable Bond will increase in price but not as much as a comparable option-free bond
- Price of Puttable bond = Price of Option-free bond + Price of Put Option
- when interest rates decline, the price of the Option-free Bond will increase and the price of the Put Option will decline
- the price of the Puttable Bond may increase or decrease depending on the relative changes in the Option-free Bond and the Put Option prices

(b) The candidates should see that there are only combinations of pairs of options that will be high or low interest rate risk.

Maturity 10 or 20 years

Coupon high or low value per maturity

Embedded option or option free

| Significant Interest Rate Risk (high or low)? | | | | Investor A (high risk) or B (low risk) |
|---|---------------|---------|------------------|--|
| Bond | Maturity | Coupon | Embedded Options | |
| A | 10 years LOW | 5% HIGH | None HIGH | A (2 of 3 factors high sensitivity) |
| B | 10 years LOW | 5% High | Put Option LOW | B (1 of 3 factors high sensitivity) |
| C | 10 years LOW | 7% LOW | None HIGH | B (1 of 3 factors high sensitivity) |
| D | 10 years LOW | 7% LOW | Call Option LOW | B (0 of 3 factors high sensitivity) |
| W | 20 years HIGH | 8% HIGH | None HIGH | A (all of 3 factors high sensitivity) |
| X | 20 years HIGH | 8% HIGH | Put Option LOW | A (2 of 3 factors high sensitivity) |
| Y | 20 years HIGH | 10% LOW | None HIGH | A (2 of 3 factors high sensitivity) |
| Z | 20 years HIGH | 10% LOW | Call Option LOW | B (1 of 3 factors high sensitivity) |

Bond portfolio for Investor A – bonds A, W, X, Y

Question #7 Solution

- (a) Advantages of simulation techniques
- It permits the inclusion of path dependency in the cashflow of the instrument being valued
 - It allows for more realistic jumps in interest rates of varying amounts along a single path
 - It allows for branches that do not have to rejoin at each node
 - Easier to use when more than one factor changes at a time

Advantages of using lattice methods

- Can be used to value American style options embedded in financial instruments
- At each node of the lattice, the price of the instrument being valued is known

- (b) How many paths?
- Increased precision can be obtained with more paths, but using more paths slows the simulation process
 - Variance reduction techniques can be used to maintain accuracy in pricing calculations with fewer paths

How many periods? How long?

- Monthly time intervals are standard for mortgage-backed securities pricing, since cashflows are usually monthly
- Number of factors and parameter estimation errors
- As more factors are added can obtain closer model fits to prices of reference securities
- Credit Liquidity and Prepayment Risk
- Can add and OAS adjustment to reflect illiquidity
- Initializing the parameter
- General approach: produce a simulation that correctly prices a set of Treasury securities options on Treasury futures, and a set of interest rate caps

- (c)
- Assign cashflows to each of the nodes and determine probabilities associated with each of these cashflows
 - Discount each cashflow by sequence of short rates that give rise to it, weighing all path dependent present values generated by the probability of occurrence of the paths
 - The sum of these discounted present values is the value of the mortgage-backed securities portfolio

Question #8 Solution

Asset allocation optimization should consider both the capital market and the Investor's specific conditions.

- (a) Capital Market: Identify the capital market conditions
Prediction Procedure
Calculate the expected return, risk and correlation.

In this step, prediction procedure is always difficult
Identify the capital market conditions, we need to specify the risk/reward
It is not an easy thing. Risk-free rate may change, capital allocation line may change and the efficient frontier may not be easily found.
Additionally, the prediction model depends on inputs/assumptions.

- (b) Investor's situation: Investor's assets, liability and net worth
Investor's risk averse function
Investor's risk tolerance

In this part, investor's assets, liability and net worth may not be constant and also the Investor's risk averse function and risk tolerance may change over time.
Investor's preference could change over time. Also, the embedded options in a portfolio often make value determination difficult.

- (c) Combine the capital Market and Investor's situation together. Determine the optimal portfolio and the characteristics of the optimal portfolio.
- model outputs depend on historic data and assumptions
 - must have thorough knowledge on investors' circumstances and preferences.

Also, model needs to be constantly updated and re-optimized. There is a tradeoff between transaction cost and optimal mix.

Generally, there are four types of models:

- Integrated Asset allocation: this model follows all the steps in the above
- Strategic Asset allocation: This is not done very frequently.
It is done for calculating the benchmark portfolio under normal economic situations. Simulations are done under various portfolio mixes. Investors look at the range and choose. In this model, since simulation is done, no need for the prediction procedures. And also it assures the investor has a constant risk averse function.
- Tactical Asset allocation: it takes advantages of the inefficiencies between assets. Normally sell past winners and buy past losers. It assures the risk averse function constant
- Insured Asset allocation: the investor's risk tolerance changes dramatically when the net worth changes.

Question #9 Solution

- (a) the market price of risk is

$$\frac{E(r_M) - r_f}{\sigma_M^2} = \frac{0.07 - 0.03}{0.045} = 0.889$$

- (b) the contribution to the variance of the market portfolio for security i are

$$W_i \times \sum W_j \text{cov}(r_i \cdot r_j) \quad j = 1, \dots$$

For B:

$$\begin{aligned} W_B \text{cov}(r_B \cdot r_M) &= 0.25 \times 0.35 \times 2\% + 0.25^2 \times 12\% + 0.25 \times 0.25 \times 1\% + 0.25 \times 0.15 \times 2\% \\ &= 0.175\% + 0.75\% + 0.0625\% + 0.075\% \\ &= 1.0625\% = 0.010625 \end{aligned}$$

For D:

$$\begin{aligned} W_D \text{cov}(r_D \cdot r_M) &= 0.15 \times 0.35 \times 8\% + 0.15 \times 0.25 \times 2\% + 0.15 \times 0.25 \times 5\% + 0.15^2 \times 12\% \\ &= 0.42\% + 0.075\% + 0.1875\% + 0.27\% \\ &= 0.9525\% = 0.009525 \end{aligned}$$

$$\text{For B and D all} = 0.010625 + 0.009525 = 0.02015$$

- (c) For i the expected rate of return

$$E(r_i) = r_f + B_i [E(r_M) - r_f] = r_f + \frac{\text{cov}(r_i, r_M)}{\sigma_M^2} [E(r_M) - r_f]$$

$$\text{For B: } \text{cov}(r_B, r_M) = \text{contribution for B} / W_B = 0.010625 / 0.25 = 0.0425$$

$$\begin{aligned} E(r_B) &= r_f + \frac{\text{cov}(r_B, r_M)}{\sigma_M^2} [E(r_M) - r_f] \\ &= 0.03 + \frac{0.0425}{0.045} \times 0.04 = 0.0678 = 6.78\% \end{aligned}$$

$$\text{For D: } \text{cov}(r_D, r_M) = \text{contribution for D} / W_D$$

$$\begin{aligned} &= 0.009525 / 0.15 = 0.0635 \\ E(r_D) &= r_f + \frac{\text{cov}(r_D, r_M)}{\sigma_M^2} [E(r_M) - r_f] \\ &= 0.03 + \frac{0.0635}{0.045} \times 0.04 = 8.64\% \end{aligned}$$

Question #10 Solution

- (a) Compare forward contracts to futures contracts
1. Futures contracts are more liquid
 2. Forward can be traded with dealer or broker but futures are traded in exchange
 3. Forward contracts can be customized in size, asset type, delivery date and delivery price but futures contracts are standardized
 4. Forward contracts realize gain or loss at delivery while futures contract, are mark-to-market where gain or loss are realized everyday. Futures contract have margin call if maintenance margin is too low.
 5. Forward contracts are subject to counterparty risk but futures contracts, have clearing house delinquency
 6. Forward have bid-ask spread but futures need commission or fee
 7. Futures do not always provide an optimal hedge
 8. For forwards, may require a good credit history or deposit as back-up from other party.

- (b) 1-month T-bill on Feb 1 2004 quoted spot price = $(1 - \text{annualized discount yield}) \times 100 = 98.5$

4-month T-bill on Feb 1 2004 = 98

$$\text{Implied price of futures} = \frac{98}{98.5} = 99.49$$

- (c) On Feb 1 2004, buy 1-month T-bill with par = 97.5
and sell short 4-month T-bill with par = 100

Go long futures contract at 97.5

$$\text{Cash flows on Feb 1, 2004} = -97.5 \left(\frac{98.5}{100} \right) + 100 \frac{98}{100} = 1.9625$$

Cash flows of March 1, 2004 = $97.5 - 97.5 = 0$

because 1-month T-bill matures with proceeds to buy the futures contracts

Cash flows on April 1, 2004 = 0

Cash flows on May 1, 2004 = $100 - 100 = 0$

because 4-month T-bill matures and is repaid by the redemption of 3-month T-bill under futures contract

Profit = 1.9625

Question #11 Solution

$$(a) \quad S = \frac{\eta S_u + (1-\eta) S_d}{1+r} = \frac{50\% (105) + (1-50\%) \cdot 95}{1+5\%} = 95.23809524$$

$$(b) \quad \text{market price of risk } \lambda = \frac{\mu_s - \gamma}{\sigma}$$
$$\sigma^2 = p(1-p)(S_u - S_d)^2 / S^2$$
$$= p(1-p)(105 - 95)^2 / (95.23809524)^2$$
$$\Rightarrow p(1-p) = \frac{(95.23809524\sigma)^2}{(105 - 95)^2} = \frac{(95.23809524)^2 \times 0.2755\%}{10^2}$$
$$= 0.249886221$$
$$\Rightarrow p - p^2 = 0.249886221 \Rightarrow p^2 - p + 0.249886221 = 0$$
$$\Rightarrow p = \frac{1 \pm \sqrt{1 - 4 \times 0.249886221}}{2} = 0.510666724$$
$$\mu_s = [pS_u + (1-p)S_d - S] / S = [p105 + (1-p)95 - S] / S$$
$$= (10p + 95 - S) / S = 0.051120006$$
$$= \frac{\mu_s - 5\%}{\sqrt{0.2755\%}} = 0.0213328287$$

$$(c) \quad \text{true probability} = 0.510666724$$

need to know that the true price and the martingale price are the same

Question #12 Solution

(a) Macauley duration = $\frac{1}{A} \sum \frac{tA_t}{(1+y)^t}$

$$A = \sum \frac{A_t}{(1+y)^t} = \frac{7.5}{1.055^{\frac{1}{2}}} + \frac{107.5}{1.055^{\frac{3}{2}}} = 106.5060208$$
$$= \frac{1}{A} \left(\frac{0.5(7.5)}{1.055^{\frac{1}{2}}} + \frac{1.5(107.5)}{1.055^{1.5}} \right)$$
$$= 1.431441568$$

(b)

Immunization

- ensures a positive surplus at the end of holding period
- 3 conditions: 1) PV (assets) > PV (liabilities)
2) Duration (assets) = Duration (liabilities)
3) Asset cashflows more disperse than liability cashflows
- often need rebalancing; portfolio not immunized after asset default, liability cashflow mature, etc.
- assure flat yield curve
- small, parallel, instantaneous interest rate shifts
- asset and liability discounted at same discount factor
- use effective duration, convexity if option embedded securities

Cash flow matching

- tries to match timing and amount of asset & liability cashflows as closely as possible to achieve a net accumulated cash flow
- limited reinvestment risk
- hard to exactly match
- if successful, eliminates reinvestment and int rate risk
- limited asset selection
- doesn't take into account option being exercised

$$(c) \quad A = PV(A) = \frac{110}{1.05^{\frac{1}{2}}} = 107.349008 \quad D = \frac{1}{A} \left(\frac{0.5 \times 110}{1.05^{\frac{1}{2}}} \right) = 0.5$$

$$C = \frac{1}{A} \sum \frac{t(t+1)A_t}{(1+y)^{t+2}} = 0.680272109$$

$$E = PV(E) = \frac{4}{1.065^{0.5}} + \frac{4}{1.065^{1.5}} + \frac{4}{1.065^{2.5}} + \frac{4}{1.065^{3.5}} + \frac{104}{1.065^{4.5}} = 92.47730506$$

$$D = \frac{1}{A} \sum \frac{tA_t}{(1+y)^t} = 4.105678444$$

$$C = \frac{1}{A} \sum \frac{t(t+1)A_t}{(1+y)^{t+2}} = 19.40893555$$

$$\text{liability: } PV = \frac{25}{1.05^{\frac{1}{2}}} + \frac{20}{1.055^{1.5}} + \frac{15}{1.061^{2.5}} + \frac{10}{1.0625^{3.5}} + \frac{5}{1.065^{4.5}} = 67.64445196$$

let w = weight in A, match duration and check PV, dispecion

$$D_p = \sum w_i D_i \text{ -duration}$$

w-weights

i- i^{th} security

$$C_p = \sum w_i C_i \text{ -convexity}$$

$$0.5w + 4.105678444(1-w) = 1.75$$

$$w = 0.653324605 \quad \therefore 65.33\% \text{ in A, } 34.67\% \text{ in E } \therefore \text{ is an immunized portfolio}$$

$$PV(\text{assets}) = 107.349008w + 92.47730506(1-w) = 102.1933545 > PV(\text{liability})$$

$$\text{convexity of assets} = 0.680272109w + 19.40893555(1-w) = 1.73 > C(\text{liab}) = 5.5$$

(d) Match last liability of first

$$2009: \text{ need } \frac{5}{104} \text{ of bond E}$$

$$2008: \text{ need } \frac{10 - \frac{5}{104} \times 4}{105} = \frac{17}{182} \text{ of bond D}$$

$$2007: \text{ need } \frac{15 - \frac{5}{104}x^4 - \frac{17}{182}x^5}{106.5} = 0.134654078 \text{ of bond C}$$

$$2006: \text{ need } \frac{20 - \frac{5}{104}x^4 - \frac{17}{182}x^5 - 0.134654078x^{6.5}}{107.5} = 0.171771236 \text{ of}$$

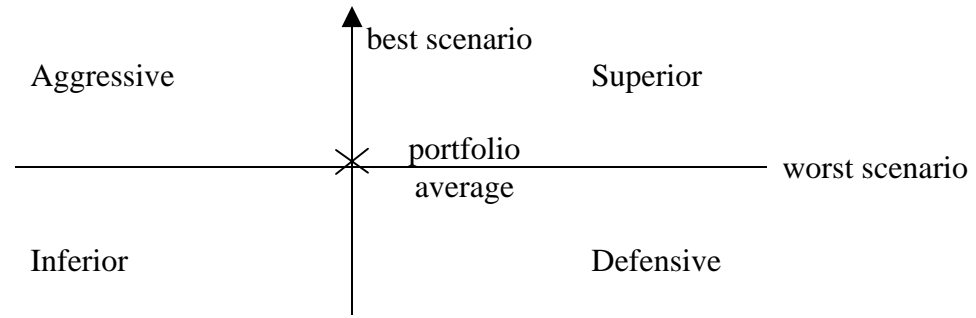
bond B

$$2005: \text{ need } \frac{20 - \frac{5}{104}x^4 - \frac{17}{182}x^5 - 0.134654078x^{6.5} - 0.171771236x^{7.5}}{110} = 0.156155669$$

of bond A

Question #13 Solution

- (a) Strategic Frontier Analysis:
- used to evaluate bonds risk characteristics
 - graph is split into quadrants
 - portfolio average is at center of graph's quadrants



Aggressive: high return in best scenario
low return in worst scenario

Superior: high return in best and worst scenarios
want to hold these bonds

Inferior: low return in best and worst scenarios
sell these bonds

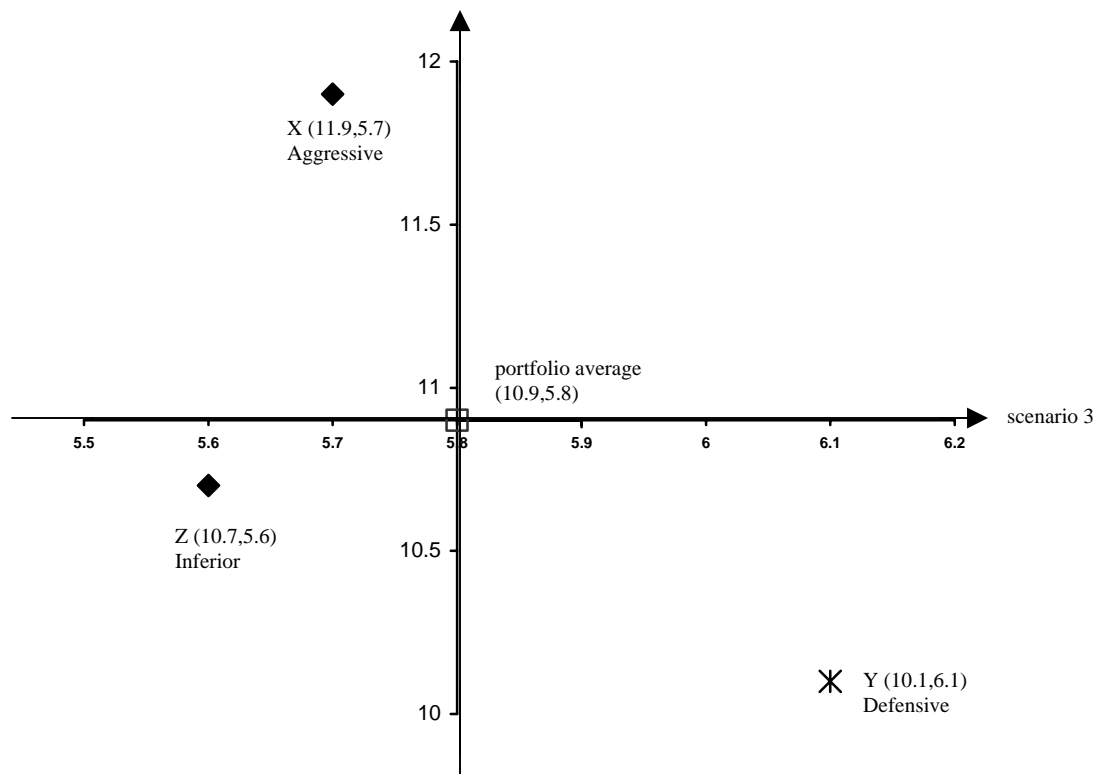
Defensive: high return in worst scenario
low return in best scenario

(b) Scenario #1 average = $\frac{1}{3}(11.9+10.1+10.7) = 10.9$

Scenario #2 average = $\frac{1}{3}(8.2+8.4+8.3) = 8.3$

Scenario #3 average = $\frac{1}{3}(5.7+6.1+5.6) = 5.8$

Scenario 1 is best scenario and scenario 3 is worst scenario



- (c) Relative Return Value Analysis:
- plot composite expected return vs. duration
 - include regression line
 - bonds above regression line have more return per unit of duration than those below regression line

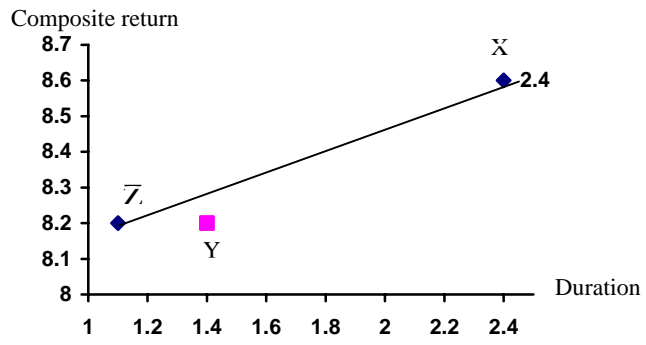
(d) Bond x return = $\frac{1}{3}(11.9 + 8.2 + 5.7) = 8.6$

Bond y return = $\frac{1}{3}(10.1 + 8.4 + 6.1) = 8.2$

Bond z return = $\frac{1}{3}(10.7 + 8.3 + 5.6) = 8.2$

Total expected return = $\frac{1}{3}(8.6 + 8.2 + 8.2) = 8.3$

Total expected duration = $\frac{1}{3}(2.3 + 1.4 + 1.1) = 1.6$



Bonds x and z are above regression line and have higher return per unit of duration than bond y.

Question #14 Solution

(a)

i.
$$r_{bd} = \left(\frac{Par - Price}{Par} \right) \left(\frac{360}{n} \right) \quad n = \text{days to maturity}$$

$$0.045 = \frac{1000 - P}{1000} \left(\frac{360}{110} \right)$$

$$Price = 986.25$$

ii.
$$r_{bey} = \left(\frac{Par - Price}{Price} \right) \left(\frac{365}{110} \right)$$

$$= \left(\frac{1000 - 986.25}{986.25} \right) \left(\frac{365}{110} \right)$$

$$= 4.63\%$$

iii.
$$r_{eay} = \left(\frac{Par}{Price} \right)^{\frac{365}{n}} - 1$$

$$= \left(\frac{1000}{986.25} \right)^{\frac{365}{110}} - 1$$

$$= 4.70\%$$

(b) T-Bills zero coupon, sold at discount
 maturity less than or equal to 1 year

T Notes semi annual coupon
 maturity 1-10 years

T Bonds Semi annual coupon
 maturity greater than 10 years

Considered a risk free security

Used as a benchmark for other securities

highly liquid

backed by full faith and credit of the US government

Sold by single price auction

competitive bids made in terms of yield

non competitive bids also can be made

All accepted bids award the lowest yield accepted (the stopout yield)

Secondary market is a multi-dealer over the counter market

highly liquid

(c) Agency Securities

Issued by agencies that all are part of or sponsored by the US Government

Low credit risk

Interest exempt from local and state taxes for most issuers

Distributed through dealers