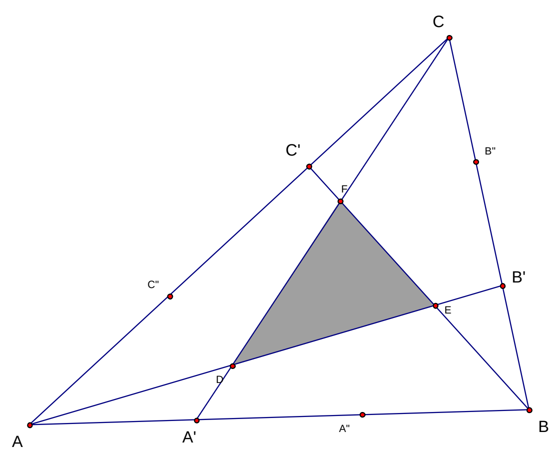
Constructions and Conjectures Project #1

With a partner or two (no more than 3 in a group):

- Create and explore the following constructions.
- Develop one solid and well-written conjecture for each construction.
- Justify **ONE** of your conjectures.
- Turn in 3 diagrams, 3 conjectures and 1 proof no later than **Sept. 22**.

1. I don't know if this figure is attributed to anyone in particular, but the results are interesting and the proof is not very difficult.
   a. Construct ΔABC (make it big enough to work with, but small enough so you have room to do some trisecting).
   b. Trisect the sides of triangle ABC.
   c. As you travel around the triangle from A to B to C, label your trisection points as follows: the first point between A and B (closest to A) is A' and the second is A". The first point between B and C (closest to B) is B' and the second is B". The first point between C and A (closest to C) is C' and the second is C".
   d. Construct segment AB', BC', and CA'.
   e. You should have an interior triangle. Label it ΔDEF.

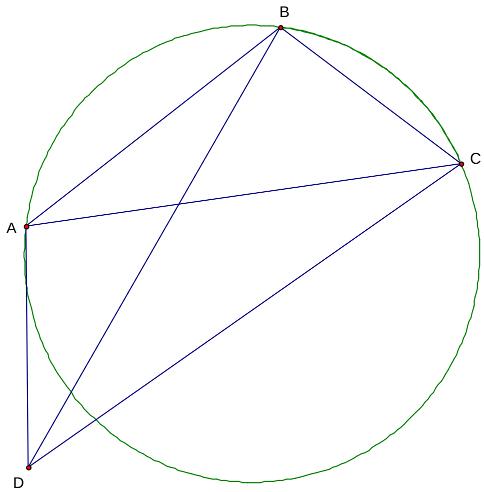
   What relationships, if any, can you find between ΔABC and ΔDEF? Consider side lengths, areas, altitudes, perimeters, etc. When you find a relationship, be sure to state it as a conjecture.



2. Ptolemy's Theorem: What you (hopefully) will discover through this exploration is another interesting relationship within cyclic quadrilaterals.
   a. Construct ΔABC.
   b. Construct the circumcircle of ΔABC by using the "Arc Through Three Points" command in the **Construct** menu.
   c. Choose a point D at random (not on the circle) and construct segments AD, BD and CD.
   d. Measure all six segments in your construction. Calculate under the **Measure** menu to display the value

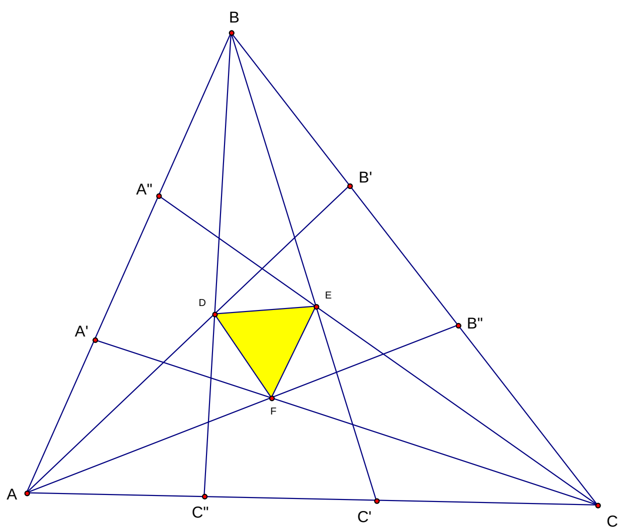
X = AB*CD + BC*AD – AC*BD

Observe the value of X as you drag point D around. Will it ever be negative? What happens when point D is on arc AC? What do you think Ptolemy's Theorem for cyclic quadrilaterals might be? Write out the theorem and then justify it. Hint: find point E on AC such that m<ABE = m<CBD, then use similar triangles.



3. Morley's Triangle: This problem is just for exploration and conjecture. The proof of it is much too complex at this point in the course, but the results are interesting, nonetheless.
   a. Construct ΔABC.
   b. Trisect each of the angles of ΔABC and locate the points where the trisecting rays intersect the sides of the triangle. Use the same labeling conventions as for construction #1, above.
   c. Let D be the intersection of AB' and BC". Let E be the intersection of BC' and CA". Let F be the intersection of CA' and AB".

What do you notice about ΔDEF?



(Sharon McCrone, MAT 211, Fall 2006)