Constructions and Conjectures Project #3

With a partner or two (no more than 3 in a group):

- Create and explore the following problems or constructions.
- Develop one solid and well-written conjecture for each construction.
- Justify **ONE** of your conjectures for #1 or #2 (#3 is not required).
- Turn in 3 sets of diagrams, 3 or more conjectures and 1 proof no later than **November 29**.

1. A Bridge Problem

A road and bridge combination is being planned to join two towns, Abington and Barnstable, separated by a river. Where should the bridge be built so the total distance across the bridge and along the straight roadway between the towns can be minimized?

To simplify the problem, assume the river banks are straight parallel lines and that the bridge is built perpendicular to the river banks.

a. Use Geometer's Sketchpad to model the situation and to explore a strategy for building the roads and bridge.
b. Does it matter where the towns are located? In other words, will a particular pair of towns yield a different solution for the location of the bridge? (Is there more than one case to consider?)
c. Develop a conjecture or set of conjectures for where to construct the bridge.
d. Give a proof of the conjecture.

Be sure to check and print several cases to support your conjecture. In other words, drag the towns and/or bridge a few times to be sure the conjecture holds in general. ***Turn in several sketches, your conjecture, and a proof of the conjecture (if you choose).***

2. The Fundamental Theorem of Isometries

The Fundamental Theorem of Isometries states that *every isometry in the plane is the composition of at most three reflections, exactly two if the isometry is direct (and nontrivial).*

This means that if you have a translation or rotation, you should be able to find two lines of reflection that take the original figure to its image under the translation or rotation. In this problem you will find a way to locate those reflection lines using just the original figure and its image.

The Translation:

a. Use Sketchpad to construct a scalene triangle ΔABC and translate it along an arbitrary vector $\overrightarrow{XY}$ to get ΔA'B'C'.

b. Hide vector $\overrightarrow{XY}$. You should not need/use to use it for the rest of the constuction.
c. Find a method for constructing a set of parallel mirror lines *l* and *m* such that the composition of two reflections over *l* and *m* will take ΔABC to ΔA'B'C'.
d. Record the steps for constructing these parallel lines.
e. Can you find a second set of lines that are different from the first? Or are those the only two lines that will work?
f. Write a conjecture that tells how to locate the parallel lines of reflection for a given translation.

The Rotation:

g. Use Sketchpad to construct a scalene triangle ΔDEF and rotate it about a point O under a set angle measure θ to get ΔD'E'F'. (You decide where to place the center and the value of θ. Be sure it is clearly marked.)
h. Hide the center of rotation (and the angle of rotation is you constructed one).
i. Find a method for constructing a set of intersecting mirror lines p and r such that the composition of two reflections over p and r will take ΔDEF to ΔD'E'F'.
j. Record the steps for constructing the intersecting mirror lines.
k. Is there only one set of lines that will work?
l. Write a conjecture that tells how to locate the intersecting lines of reflection for a given rotation.

Justify one of your conjectures (translation or rotation). *Be sure to hand in your constructions, the steps for locating the lines for the Translation and the Rotation, and a conjecture for each. Choose one of the conjectures and justify that you have correctly located the lines (if you choose).*

3. <u>A Buried Treasure Problem</u>

The island is uninhabited, relatively flat, and has only two palm trees (one short and one tall) to serve as landmarks. Find a suitable place on the island where you can see both palm trees and drive a stake. Proceed to the taller tree, counting your paces. When you get there, turn 90° to the left, and walk an equal number of paces. At that point drive a second stake. Return to your starting point and proceed to the other tree, counting your paces. When you come to the second tree, turn 90° to the right and walk an equal number of paces. Drive a third stake at this point. Now the treasure lies midway between the second and third stakes.

Is there a flaw in this treasure "map?" No starting point is specified so you could begin at just about any point on the island. How will you know if you've chosen the right starting point in order to find the buried treasure.

a. Simulate the directions on Sketchpad and see what happens.
b. What starting point might be the correct starting point?
c. Write a conjecture related to your findings.
d. Can you find a justification for your conjecture? (Not required)

Be sure to hand in a conjecture along with sketches for several different starting points to support your conjecture. If you wish to attempt a justification or explanation, that would be great but is ***not required****.*

(Sharon McCrone, MAT 211, Fall 2006)