Constructions and Conjectures Project #2

With a partner or two (no more than 3 in a group):

- Create and explore the following constructions.
- Develop one solid and well-written conjecture for each construction.
- Justify **TWO** of your conjectures.
- Turn in 3 sets of diagrams, 3 or more conjectures and 2 proofs no later than **October 20**.

1. Cyclic Quadrilaterals, Again!

It can be easily shown that in Euclidean geometry all triangles can be circumscribed (in fact we proved this in the lab). This is not true for quadrilaterals. Those quadrilaterals that can be circumscribed are called **cyclic**.

Suppose Quadrilateral BCDE is a cyclic quadrilateral circumscribed by the circle centered at point A.

a. Use Geometer's Sketchpad to draw a cyclic quadrilateral BCDE circumscribed by the circle with center A.
b. Now, using the **Transform** menu, reflect point A over each of the sides of quadrilateral BCDE (To reflect a point over a segment or line you need to mark the line of reflection or "mirror" first. Then highlight point A and choose "reflect" from the Transform menu).
c. Let the points of reflection be points W, X, Y and Z. Connect the 4 points of reflection to construct the new quadrilateral WXYZ.
d. Investigate this new quadrilateral and state a conjecture about it.
e. Give a proof of the conjecture.

Be sure to check and print several cases to support your conjecture. In other words, drag the quadrilateral a few times to be sure the conjecture holds for a general cyclic quadrilateral. Turn in several sketches, your conjecture, and a proof of the conjecture (if you choose).

2. Orthic Triangles

An *orthic triangle* is formed by constructing the altitudes from each vertex of a triangle. The foot of each altitude is a vertex of the orthic triangle.

a. Use Geometer's Sketchpad to construct ΔABC and orthic triangle ΔDEF for various types of triangles (e.g., right triangle, obtuse triangle, etc.).
b. Now compute the ratio of areas:

$$\frac{\text{Area of Orthic Triangle}}{\text{Area of Original Triangle}}$$

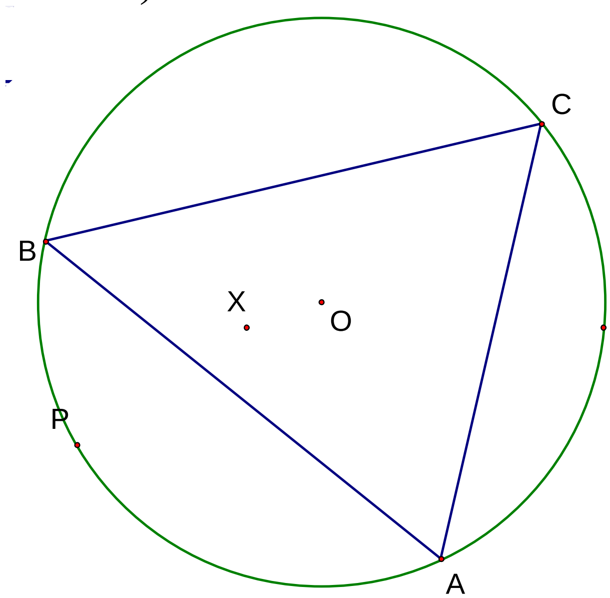
c. Drag the original triangle and investigate what happens to the ratio. What can you say about the ratio in terms of possible maximum and minimum values?

d. Test different kinds of triangles: acute, obtuse, right, equilateral, isosceles. For which can you determine a maximum and/or minimum value? What are the maximum and minimum values?
e. Choose one result (max and min for one type of triangle) from your investigations in part (d) to prove.

Be sure to hand in your results for each type of triangle you test. In other words, you should submit sketches for various types of triangles that include calculations of the ratios and a statement about possible maximum and minimum values. On a separate page, provide a conjecture for the maximum and minimum values of the ratio above for one type of triangle and prove your conjecture (if you choose).

3. Start with ΔABC and its circumcircle centered at point O.
   a. Let P be an arbitrary point on the circumcircle.
   b. For each position of point P on the circumcircle, locate point X on segment PC such that PX = PB.
   c. Construct the locus of point X as P varies around the circle. What do you notice?
   d. Now measure angle BXC and watch what happens as P moves along arc AB. What changes when you change the shape of the original triangle? Can you make any conjectures?
   e. This time start with an equilateral triangle ΔABC. What do you notice about ΔPXB as P travels along arc AB? Write a conjecture and prove it.

Be sure to hand in your sketches and results for a couple types of triangles for parts (c) and (d). For part (e) you should have a sketch, a conjecture and a proof (if you choose).



(Sharon McCrone, MAT 211, Fall 2006)