

The Curve-Drawing Machines of Descartes

The co-founders of *analytic geometry*, Pierre de Fermat (1601-1665) and René Descartes (1596-1650), had different approaches to combining *geometry* and *algebra* (a.k.a. the “*analytic art*”; today “analysis” means Calculus and beyond). Descartes viewed curves as the primary objects of study (in line with old Greek tradition), and tried to define what kind of geometric figures were worthy of the name “curve”, apart from lines and circles. He made the cryptic announcement that “[t]wo or more lines can be moved, one upon the other, determining by their intersection other curves”. That is, legitimate curves could only be drawn by (real or imaginary) mechanical devices involving movable arms. This is far from being a well-defined concept! When he tried to describe his curves via algebra (relationships of the coordinates), however, he ran into some complicated equations.

On the other hand, Fermat started with algebraic equations, and then considered the curves represented by these equations. He had the advantage of being able to begin with rather simple equations. Both men made substantial contributions to mathematics and were among the most prominent mathematicians who laid the groundwork for Calculus. Over the centuries, we seem to have lost the geometric intuition... we don't really care how a parabola is drawn (that's what graphing calculators are for) but we do know that the algebraic equation that describes the points (x, y) on an upright parabola is of the form $y = ax^2 + bx + c$. Progress?

Problem: Find out the descriptions of some of Descartes' curve-drawing machines and construct them either as actual mechanical devices or as imaginary ones with the software *Geometer's Sketchpad*. Explain analytically why these gadgets trace the curves Descartes claimed they did.

Reference: *A History of Mathematics: An Introduction* (2nd edition) by V.J. Katz (Addison Wesley Longman, 1998)