Undergraduate Colloquium in Mathematics
MAT268 Student Projects

Wednesday May 2, 2018 3:00PM STV 324

The students from MAT 268 *Introduction to Undergraduate Research in Mathematics* worked in groups of 3 or 4 on 3 projects in Combinatorial Design Theory. They were mentored by Saad El-Zanati and Ryan Bunge. Each group will give a 15 minute presentation on their results.

On the Spectrum Problem for the Orientations of the 6-Cycle

Jordan Dulowski, Maddie Kenney, Katie Zale

Graphs are mathematical means of exhibiting relationships between objects. The objects are denoted by points, called *nodes* or *vertices*, and each pair of related objects is joined by a line, called an *edge*. For example, friendships on social media can be represented by friendship graphs. People are denoted by nodes and two nodes are joined by an edge if the corresponding people are friends on Facebook. If the relationships are directional, the edges are directed to reflect this property. For example, if Person A follows Person B on Twitter, the edge between them (now called an *arc*) is oriented from node A to node B. If both A and B follow each other, the corresponding nodes are joined by arcs in both directions. The *complete symmetric digraph* of *order* n, denoted by K_n^*, is the directed graph on n nodes with the property that every pair of nodes is joined by an arc in each direction. A *6-cycle* is a graph with 6 nodes, say A, B, C, D, E, and F, such that A and B are related, and so are B and C, C and D, D and E, E and F, and F and A. An *orientation* of a 6-cycle is an assignment of a direction to each of the edges. A common problem in the study of graphs is the problem of deciding when a large graph can be partitioned (i.e., divided up) into pieces that all have the exact same structure as some smaller graph G. The *spectrum problem* for a given directed graph G is the problem of determining the values of n so that K_n^* can be partitioned into copies of G. This problem has been studied and settled for the orientations of 3-, 4-, and 5-cycles. We settle it for the 9 orientations of a 6-cycle.

On the Spectrum Problem for the Connected Cubic Bipartite Multigraphs of Order 8

Katie Battista, William Duncan, Colleen Hehr, Tracer Mills

In a *multigraph*, multiple edges are allowed between pairs of nodes. The class of complete multigraphs is of particular interest. The *λ-fold complete graph* of *order n*, denoted by ${}^\lambda K_n$, is the graph on n nodes with the property that every pair of nodes is joined by λ edges. A multigraph G is *cubic* if every node in G has 3 edges joined to it. The spectrum problem for a given multigraph G is the problem of determining the values of n so that ${}^\lambda K_n$ can be partitioned into copies of G. This problem has been studied and settled for cubic multigraphs on less than 8 nodes. We study the $\lambda = 2$ version for the cubic connected multigraphs on 8 nodes.

On Decompositions of Complete 3-Uniform Hypergraphs into Loose 4-Cycles

Lauren Haman, Cody Hatzer, Kristen Koe, Kayla Spornberger

Hypergraphs generalize the concept of a graph. In a k-uniform hypergraph, edges are represented by sets of k nodes rather than by pairs of nodes. The *complete k-uniform hypergraph* of *order n*, denoted by $K_n^{(k)}$, is the k-uniform hypergraph on n nodes with the property that every set of k nodes is joined by an edge. A common problem in the study of graphs is the problem of deciding when a large graph or hypergraph can be partitioned (i.e., divided up or decomposed) into pieces that all have the exact same structure as some smaller (hyper)graph. A *loose 4-cycle* is a 3-uniform hypergraph with vertex set $\{a, b, c, d, e, f, g, h\}$ and edge set $\{\{a, b, c\}, \{c, d, e\}, \{e, f, g\}, \{g, h, a\}\}$. We find necessary and sufficient conditions for the existence of loose 4-cycle decompositions of $K_n^{(3)}$.