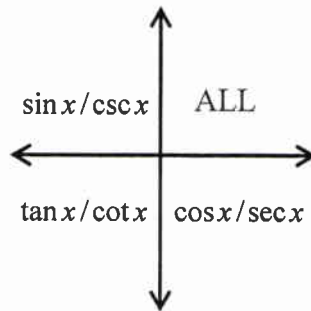


Trigonometry

Trig Values

$\sin 0 = 0$	$\cos 0 = 1$	$\tan 0 = 0$
$\sin \frac{\pi}{6} = \frac{1}{2}$	$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$	$\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$
$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$	$\tan \frac{\pi}{4} = 1$
$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$	$\cos \frac{\pi}{3} = \frac{1}{2}$	$\tan \frac{\pi}{3} = \sqrt{3}$
$\sin \frac{\pi}{2} = 1$	$\cos \frac{\pi}{2} = 0$	$\tan \frac{\pi}{2}$ DNE
$\sin \pi = 0$	$\cos \pi = -1$	$\tan \pi = 0$
$\sin \frac{3\pi}{2} = -1$	$\cos \frac{3\pi}{2} = 0$	$\tan \frac{3\pi}{2}$ DNE
$\sin 2\pi = 0$	$\cos 2\pi = 1$	$\tan 2\pi = 0$

Positive Trig Values



Transformations of Trig Graphs

(See next page for basic trig graphs)

$$y = A \sin(Bx + C) + D \text{ or } y = A \cos(Bx + C) + D$$

Amplitude is A units

Period is $\frac{2\pi}{B}$

Phase shift is $\frac{C}{B}$ units (right if $C < 0$; left if $C > 0$)

Vertical shift is D units (down if $D < 0$; up if $D > 0$)

$$y = A \tan(Bx + C) + D$$

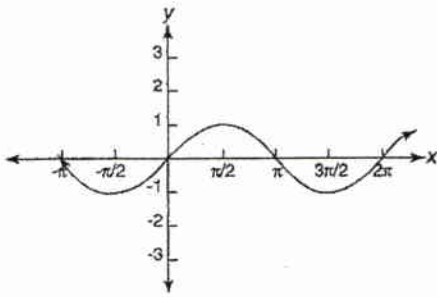
$A \neq \pm 1$ causes a vertical stretch or shrink

Period is $\frac{\pi}{B}$

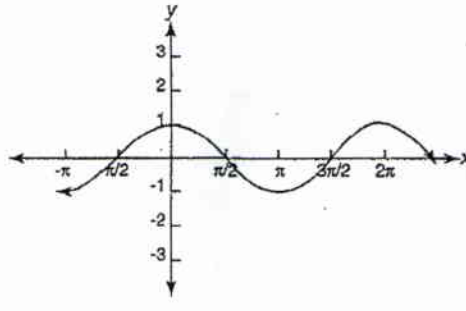
Phase shift is $\frac{C}{B}$ units (right if $C < 0$; left if $C > 0$)

Vertical shift is D units (down if $D < 0$; up if $D > 0$)

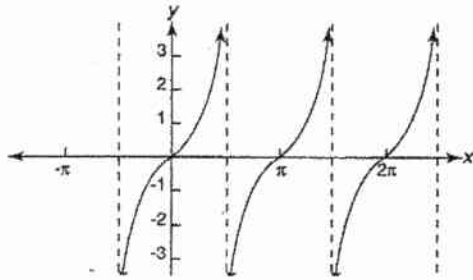
Trig Graphs



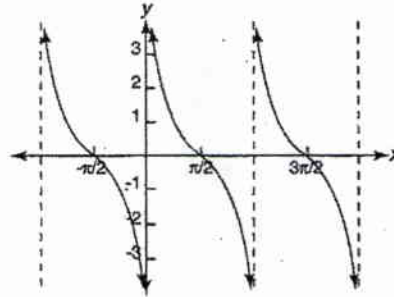
$y = \sin x$
 D : all real numbers
 R : $-1 \leq y \leq 1$
 Period: 2π



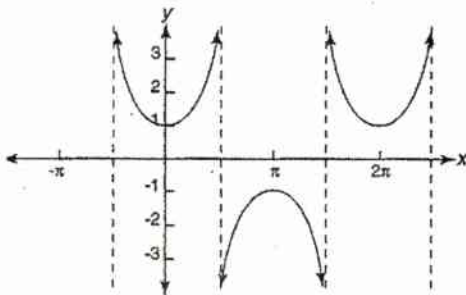
$y = \cos x$
 D : all real numbers
 R : $-1 \leq y \leq 1$
 Period: 2π



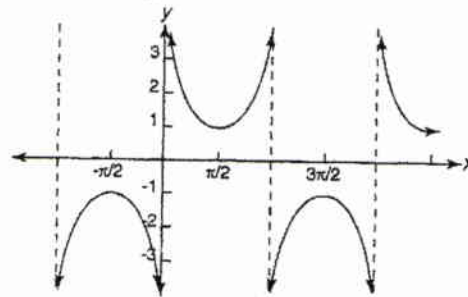
$y = \tan x$
 D : $x \neq \pi/2 + k\pi$
 R : all real numbers
 Period: π



$y = \cot x$
 D : $x \neq k\pi$
 R : all real numbers
 Period: π



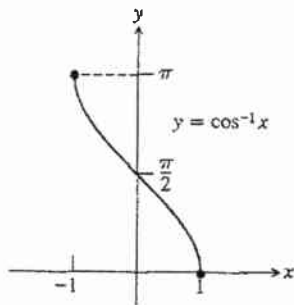
$y = \sec x$
 D : $x \neq \pi/2 + k\pi$
 R : $y \leq -1$ or $y \geq 1$
 Period: 2π



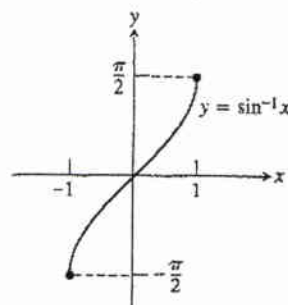
$y = \csc x$
 D : $x \neq k\pi$
 R : $y \leq -1$ or $y \geq 1$
 Period: 2π

Inverse Trig Graphs

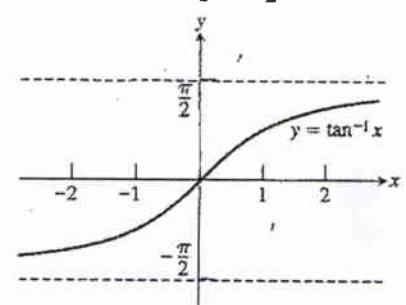
Domain: $-1 \leq x \leq 1$
 Range: $0 \leq y \leq \pi$



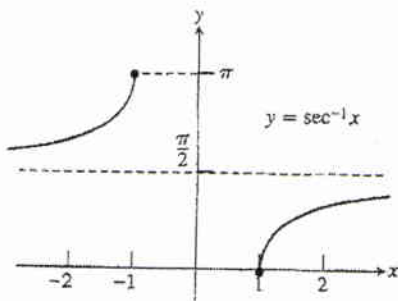
Domain: $-1 \leq x \leq 1$
 Range: $-\pi/2 \leq y \leq \pi/2$



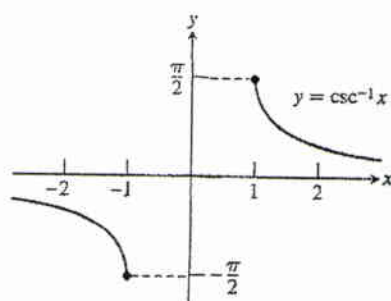
Domain: $-\infty < x < \infty$
 Range: $-\pi/2 < y < \pi/2$



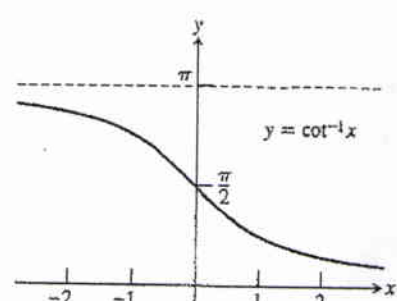
Domain: $x \leq -1$ or $x \geq 1$
 Range: $0 \leq y \leq \pi, y \neq \pi/2$



Domain: $x \leq -1$ or $x \geq 1$
 Range: $-\pi/2 \leq y \leq \pi/2, y \neq 0$



Domain: $-\infty < x < \infty$
 Range: $0 < y < \pi$



Trigonometric Identities

$$1.) \sin^2 x + \cos^2 x = 1$$

$$2.) 1 + \tan^2 x = \sec^2 x$$

$$3.) 1 + \cot^2 x = \csc^2 x$$

$$4.) \sin(-x) = -\sin x$$

$$5.) \cos(-x) = \cos x$$

$$6.) \tan(-x) = -\tan x$$

$$7.) \sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$8.) \sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$9.) \cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$10.) \cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$11.) \sin(2x) = 2 \sin x \cos x$$

$$12.) \cos(2x) = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$a. \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos(2x)$$

$$b. \sin^2 x = \frac{1}{2} - \frac{1}{2} \cos(2x)$$

$$13.) \tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}$$

$$14.) \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$15.) \csc x = \frac{1}{\sin x}$$

$$16.) \sec x = \frac{1}{\cos x}$$

$$17.) \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$18.) \sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$19.) \sin A \sin B = \frac{1}{2} \cos(A - B) - \frac{1}{2} \cos(A + B)$$

$$20.) \cos A \cos B = \frac{1}{2} \cos(A - B) + \frac{1}{2} \cos(A + B)$$

$$21.) \sin A \cos B = \frac{1}{2} \sin(A - B) + \frac{1}{2} \sin(A + B)$$

Miscellaneous Limit Rules

Constant Rules

- 1.) $\lim_{x \rightarrow a} c = c$
- 2.) $\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$

Limit Rules

- 3.) $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
- 4.) $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
- 5.) $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
- 6.) $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}, \quad \lim_{x \rightarrow a} g(x) \neq 0$

Limits to Infinity (for polynomial functions $f(x)$ & $g(x)$)

- 7.) $\lim_{x \rightarrow \pm\infty} \left[\frac{f(x)}{g(x)} \right] = 0$ if the degree of $f(x)$ < the degree of $g(x)$. For example, $\lim_{x \rightarrow \infty} \frac{x^2 - 2x}{x^3 + 3} = 0$
- 8.) $\lim_{x \rightarrow \pm\infty} \left[\frac{f(x)}{g(x)} \right]$ is infinite if the degree of $f(x)$ > the degree of $g(x)$. The limit will be ∞ or $-\infty$.
Examples: $\lim_{x \rightarrow -\infty} \frac{x^3 - 2x}{x^2 + 3} = -\infty$ $\lim_{x \rightarrow \infty} \frac{x^3 - 2x}{x^2 + 3} = \infty$ $\lim_{x \rightarrow -\infty} \frac{x^3 - 2x}{3 - x^2} = \infty$
- 9.) $\lim_{x \rightarrow \pm\infty} \left[\frac{f(x)}{g(x)} \right] = c$ if the degrees of $f(x)$ and $g(x)$ are equal. The value of c is found by dividing the leading coefficient of $f(x)$ by the leading coefficient of $g(x)$. For example, $\lim_{x \rightarrow -\infty} \frac{2x^2 - 3x + 2}{10x - 5x^2} = -\frac{2}{5}$

Trig Limits

- 10.) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$
- 11.) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$

The number e as a limit

- 12.) $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x = e$
- 13.) $\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e$

Differentiation Formulas

(c is a constant)

$$1.) \frac{d}{dx}(c) = 0$$

$$2.) \frac{d}{dx}(x^n) = nx^{n-1}$$

$$3.) \frac{d}{dx}(cf(x)) = cf'(x)$$

$$4.) \frac{d}{dx}(f(x) \cdot g(x)) = f(x)g'(x) + g(x)f'(x) \text{ (Product Rule)}$$

$$5.) \frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \text{ (Quotient Rule)}$$

$$6.) \frac{d}{dx}(\sin x) = \cos x$$

$$7.) \frac{d}{dx}(\cos x) = -\sin x$$

$$8.) \frac{d}{dx}(\tan x) = \sec^2 x$$

$$9.) \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$10.) \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$11.) \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$12.) \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$13.) \frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$14.) \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$15.) \frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

$$16.) \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$17.) \frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$18.) \frac{d}{dx}(e^x) = e^x$$

$$19.) \frac{d}{dx}(a^x) = a^x \ln a$$

$$20.) \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$21.) \frac{d}{dx}(\ln x) = \frac{1}{x}$$

THE CHAIN RULE:

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

****You MUST apply the chain rule when using rules 1 – 21 if the “x” is replaced by ANY OTHER FUNCTION!**

Integration Formulas
(a and C are constants)

$$1.) \int a dx = ax + C$$

$$2.) \int x^a dx = \frac{x^{a+1}}{a+1} + C$$

$$3.) \int \frac{1}{x} dx = \ln|x| + C$$

$$4.) \int e^x dx = e^x + C$$

$$5.) \int a^x dx = \frac{a^x}{\ln a} + C$$

$$6.) \int \ln x dx = x \ln x - x + C$$

$$7.) \int \sin x dx = -\cos x + C$$

$$8.) \int \cos x dx = \sin x + C$$

$$9.) \int \tan x dx = \ln|\sec x| + C$$

$$10.) \int \csc x dx = \ln|\csc x - \cot x| + C$$

$$11.) \int \sec x dx = \ln|\sec x + \tan x| + C$$

$$12.) \int \cot x dx = \ln|\sin x| + C$$

$$13.) \int \sec^2 x dx = \tan x + C$$

$$14.) \int \sec x \tan x dx = \sec x + C$$

$$15.) \int \csc^2 x dx = -\cot x + C$$

$$16.) \int \csc x \cot x dx = -\csc x + C$$

$$17.) \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$18.) \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$19.) \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$$

Basic Integral Rules

$$1.) \int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$2.) \int_a^a f(x)dx = 0$$

$$3.) \int_a^b kf(x)dx = k \int_a^b f(x)dx \text{ where } k \text{ is a constant}$$

$$4.) \int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$5.) \int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$$

$$6.) \int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

$$7.) \text{ If } f(x) \text{ is an ODD FUNCTION, then } \int_{-a}^a f(x)dx = 0$$

$$8.) \text{ If } f(x) \text{ is an EVEN FUNCTION, then } \int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx$$

$$9.) \text{ If } f(x) \geq 0 \text{ on } [a, b], \text{ then } \int_a^b f(x)dx \geq 0$$

$$10.) \text{ If } g(x) \geq f(x) \text{ on } [a, b], \text{ then } \int_a^b g(x)dx \geq \int_a^b f(x)dx$$

$$\text{If } F(x) \text{ is an antiderivative of } f(x), \text{ then } \int_a^b f(x)dx = F(b) - F(a)$$

Formulas & Theorems

1.) Limits and Continuity

A function $y = f(x)$ is continuous at $x = a$ if:

- (i) $f(a)$ is defined ($f(a)$ exists)
- (ii) $\lim_{x \rightarrow a} f(x)$ exists*
- (iii) $\lim_{x \rightarrow a} f(x) = f(a)$

Otherwise, f is discontinuous at $x = a$.

*The limit $\lim_{x \rightarrow a} f(x)$ exists if and only if both corresponding one-sided limits exist and are equal; that is,

$$\lim_{x \rightarrow a} f(x) = L \Leftrightarrow \lim_{x \rightarrow a^+} f(x) = L = \lim_{x \rightarrow a^-} f(x)$$

2.) Intermediate Value Theorem

A function $y = f(x)$ that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$

NOTE: If f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ differ in sign, then the equation $f(x) = 0$ has at least one solution in the open interval (a, b) . (In other words, $f(x)$ will have at least one zero on (a, b) .)

3.) Horizontal and Vertical Asymptotes

- (i) A line $y = b$ is a horizontal asymptote of the graph of $y = f(x)$ if either $\lim_{x \rightarrow \infty} f(x) = b$ or $\lim_{x \rightarrow -\infty} f(x) = b$
- (ii) A line $x = a$ is a vertical asymptote of the graph of $y = f(x)$ if either $\lim_{x \rightarrow a^-} f(x) = \pm\infty$ or $\lim_{x \rightarrow a^+} f(x) = \pm\infty$

4.) Average and Instantaneous Rate of Change

- (i) *Average Rate of Change:* If (x_1, y_1) and (x_2, y_2) are points on the graph of $y = f(x)$, then the average rate of change of y with respect to x over the interval $[x_1, x_2]$ is

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

- (ii) *Average Rate of Change (alternate):* The average rate of change for a function $y = f(x)$ on the interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$.

- (iii) *Instantaneous Rate of Change:* The instantaneous rate of change of $y = f(x)$ at a point on the graph where $x = a$ is $f'(a)$.

5.) Definitions of the Derivative

(i) Definition of the derivative of $f(x)$: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

(ii) Definition of the derivative of $f(x)$ at the point where $x = a$: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

(iii) Another definition of the derivative of $f(x)$ at the point where $x = a$: $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

The latter two definitions of the derivative is the instantaneous rate of change of $f(x)$ with respect to x at $x = a$. Geometrically, the derivative of a function at a point is the slope of the tangent line to the graph of the function at that point.

6.) Rolle's Theorem

If f is continuous on $[a, b]$ and differentiable on (a, b) such that $f(a) = f(b)$, then there is at least one number c in the open interval (a, b) such that $f'(c) = 0$. (Note: This can also be found using the Mean Value Theorem, #7)

7.) Mean Value Theorem

If f is continuous on $[a, b]$ and differentiable on (a, b) , then there is at least one number c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$. In other words, there will be at least one point on the interval (where $x = c$) where the tangent line will be parallel to the secant line connecting the endpoints of the interval; that is, the slope of the tangent line at c is equal to the slope of the line connecting the points where $x = a$ and $x = b$.

8.) Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then $f(x)$ has both a maximum and a minimum on $[a, b]$.

9.) First Derivative Test for Local Extrema

To find the maximum and minimum values of a function $y = f(x)$,

(i) the point(s) where $f'(x)$ changes sign. To do this, determine the points where $f'(x) = 0$ or where $f'(x)$ does not exist. (These are considered the critical points for $f(x)$)

(ii) the end points, if any, on the domain of $f(x)$.

- If $f'(x)$ changes sign from positive to negative at $x = a$, then $f(x)$ has a maximum at $x = a$.
- If $f'(x)$ changes sign from negative to positive at $x = a$, then $f(x)$ has a minimum at $x = a$.
- If $x = b$ is a LEFT endpoint and $f'(b) > 0$, then $f(x)$ has a minimum at $x = b$.
- If $x = b$ is a LEFT endpoint and $f'(b) < 0$, then $f(x)$ has a maximum at $x = b$.
- If $x = c$ is a RIGHT endpoint and $f'(c) > 0$, then $f(x)$ has maximum at $x = c$.
- If $x = c$ is a RIGHT endpoint and $f'(c) < 0$, then $f(x)$ has a minimum at $x = c$.

10.) Second Derivative Test for Local Extrema

- i.) If $f'(c) = 0$ and $f''(c) < 0$, then $f(x)$ has a local maximum at $x = c$.
- ii.) If $f'(c) = 0$ and $f''(c) > 0$, then $f(x)$ has a local minimum at $x = c$.

11.) Let f be differentiable for $a < x < b$ and continuous for $a \leq x \leq b$.

- (i) If $f'(x) > 0$ for every x in (a, b) , then f is increasing on $[a, b]$.
- (ii) If $f'(x) < 0$ for every x in (a, b) , then f is decreasing on $[a, b]$.

12.) Suppose that $f''(x)$ exists on the interval (a, b) .

- (i) If $f''(x) > 0$ in (a, b) , then f is concave upward in (a, b) .
- (ii) If $f''(x) < 0$ in (a, b) , then f is concave downward in (a, b) .

To locate the points on inflection of $y = f(x)$, find the points where $f''(x) = 0$ or where $f''(x)$ fails to exist. These are the only candidates where $f(x)$ may have a point of inflection. Test these points to make sure that $f''(x) < 0$ on one side and $f''(x) > 0$ on the other side. (i.e., $f''(x)$ must change signs at a point in order for $f(x)$ to have a point of inflection there.)

13.) Differentiability implies continuity: If a function is differentiable at a point $x = a$, it is continuous at that point. The converse is false; that is, continuity does not imply differentiability.

14.) Linear Approximation

The linear approximation of $f(x)$ near $x = a$ is given by $y = f(a) + f'(a)(x - a)$.

To estimate the slope of a graph at a point, draw a tangent line to the graph at that point. Another way is by using a graphing calculator to “zoom in” around the point in question until the graph “looks” straight. This method almost always works. If we “zoom in” and the graph looks straight at a point, say $x = a$, then the function is locally linear at that point.

15.) Inverse Functions

- (i) If f and g are two functions such that $f(g(x)) = x$ for every x in the domain of g , and $g(f(x)) = x$ for every x in the domain of f , then f and g are inverse functions of each other.
- (ii) A function f has an inverse function if and only if no horizontal line intersects its graph more than once.
- (iii) If f is either increasing or decreasing in an interval, then f has an inverse function over that interval.
- (iv) If f is differentiable at every point on an interval I , and $f'(x) \neq 0$ on I , then $g = f^{-1}(x)$ is differentiable at every point of the interior of the interval $f(I)$ and $g'(f(x)) = \frac{1}{f'(x)}$.

16.) Properties of e^x

- (i) The exponential function $y = e^x$ is the inverse function of $y = \ln x$.
- (ii) The domain is the set of all real numbers, $(-\infty < x < \infty)$.
- (iii) The range is the set of all positive numbers, $y > 0$.
- (iv) $\frac{d}{dx}(e^x) = e^x$
- (v) $y = e^x$ is continuous, increasing, and concave up for all x .
- (vi) $\lim_{x \rightarrow +\infty} e^x = +\infty$ and $\lim_{x \rightarrow -\infty} e^x = 0$
- (vii) $e^{\ln x} = x$, for $x > 0$; $\ln(e^x) = x$ for all x .

17.) Properties of $\ln x$

- (i) The domain of $y = \ln x$ is the set of all positive numbers, $x > 0$.
- (ii) The range of $y = \ln x$ is the set of all real numbers, $-\infty < y < \infty$.
- (iii) $y = \ln x$ is continuous and increasing everywhere on its domain.
- (iv) $\ln(ab) = \ln a + \ln b$
- (v) $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
- (vi) $\ln(a^r) = r \ln a$
- (vii) $y = \ln x < 0$ if $0 < x < 1$, and $y = \ln x > 0$ if $x > 1$.
- (viii) $\lim_{x \rightarrow +\infty} \ln x = +\infty$ and $\lim_{x \rightarrow 0^+} \ln x = -\infty$.
- (ix) $\log_a x = \frac{\ln x}{\ln a}$

18.) Riemann Sums

A Riemann Sum is the sum of the areas of a specified number of rectangles drawn under the graph of $y = f(x)$ on an interval $[a, b]$. The interval $[a, b]$ will be divided into n subintervals, and rectangles will be drawn using either the right endpoint of each subinterval, the left endpoint of each subinterval, or the midpoint of each subinterval.

19.) Trapezoidal Rule

If a function f is continuous on the closed interval $[a, b]$ where $[a, b]$ has been partitioned into n equal subintervals $[x_0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n]$, each of length $\frac{b-a}{n}$, then

$$\int_a^b f(x) dx \approx \frac{b-a}{2n} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)].$$

Note: If subintervals are not equal, a trapezoidal approximation can be made by finding the areas of trapezoids individually. The above formula will not work!

20.) Definition of Definite Integral as the Limit of a Riemann Sum

Suppose that a function $f(x)$ is continuous on the closed interval $[a, b]$. Divide the interval into n equal subintervals, of length $\Delta x = \frac{b-a}{n}$. Choose one number in each subinterval; i.e., x_1 in the first, x_2 in the

second, ..., x_k in the k^{th} , ..., and x_n in the n^{th} . Then $\lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x = \int_a^b f(x) dx$.

21.) Fundamental Theorem of Calculus

(i) $\int_a^b f(x)dx = F(b) - F(a)$, where $F'(x) = f(x)$

(ii) $\frac{d}{dx} \left(\int_c^x f(t)dt \right) = f(x)$

(iii) $\frac{d}{dx} \left(\int_c^{g(x)} f(t)dt \right) = f(g(x))g'(x)$

(iv) $\frac{d}{dx} \left(\int_{h(x)}^{g(x)} f(t)dt \right) = f(g(x))g'(x) - f(h(x))h'(x)$

22.) Velocity, Speed, and Acceleration

- (i) The velocity of an object tells how fast it is going *and* in which direction (vector). Velocity is the instantaneous rate of change of position.
- (ii) The speed of an object is the absolute value of the velocity, $|v(t)|$. It is a scalar that tells how fast the object is going, disregarding its direction.

The speed of a particle increases (particle speeds up) when the velocity and acceleration have the same sign. The speed decreases (particle slows down) when the velocity and acceleration have opposite sign.

- (iii) The acceleration is the instantaneous rate of change of velocity. It is the derivative of the velocity; that is, $a(t) = v'(t)$. Negative acceleration (deceleration) means that the velocity is decreasing. Positive acceleration means that the velocity is increasing. The sign of acceleration DOES NOT indicate direction of particle motion!

Therefore, if s is the position of a moving object and t is time, then:

(i) velocity = $v(t) = s'(t) = \frac{ds}{dt}$

(ii) acceleration = $a(t) = s''(t) = v'(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

(iii) $v(t) = \int a(t)dt$

(iv) $s(t) = \int v(t)dt$

(v) The displacement of an object for $a \leq t \leq b$ is $s(a) - s(b)$, or $\int_a^b v(t)dt$.

(vi) The total distance traveled by an object for $a \leq t \leq b$ is $\int_a^b |v(t)|dt$

(vii) The position of an object at any time $t = c$ is equal to its initial position plus the displacement, or (initial position) + $\int_0^c v(t)dt$.

(viii) The average velocity of an object over the time interval from t_0 to another time t is
 Average Velocity = $\frac{\text{Change in position}}{\text{Length of time}} = \frac{s(t) - s(t_0)}{t - t_0}$, where $s(t)$ is the position of the object at time t .

23.) Average Value of a Function

The average value of $f(x)$ on $[a, b]$ is $\frac{1}{b-a} \int_a^b f(x) dx$

24.) Area between Two Curves

If f and g are continuous functions such that $f(x) \geq g(x)$ on $[a, b]$, then the area between the curves is

$$\int_a^b [f(x) - g(x)] dx.$$

(Note: You should draw a segment within the region. If the segment is vertical, use the above formula. If the segment is horizontal, you will need to take what x is equal to on the right, minus what x is equal to on the left, as your integrand. You will use the y -coordinates that bound the region as your limits of integration.

25.) Volume of Solids of Revolution

Let f be nonnegative and continuous on $[a, b]$, and let R be the region bounded above by $y = f(x)$, below by the x -axis, and on the sides by the lines $x = a$ and $x = b$. When this region R is revolved about the x -axis, it generates a solid (having cross sections that are circular disks) whose volume is $V = \int_a^b \pi r^2 dx$. (In this case, $r = f(x)$.)

In general, a solid with circular disks for cross sections has volume $V = \int_a^b \pi r^2 dx$. A solid with washers for

cross sections is $V = \int_a^b \pi [R^2 - r^2] dx$, where R is the outer radius and r is the inner radius. In both cases, if the radius is vertical, the integrand will be a function of x . If the radius is horizontal, the integrand will be a function of y .

The volume of a solid of revolution may also be determined using the cylindrical shell method. The

volume will be $V = \int_a^b 2\pi r h dx$. To determine r and h , draw a segment parallel to the axis of rotation. h is

the length of the segment (a function of x if h is vertical, a function of y if h is horizontal). r is the distance between the segment and the axis of rotation.

26.) Volumes of Solids with Known Cross Sections

(i) For cross sections of area $A(x)$, taken perpendicular to the x -axis, volume is $V = \int_a^b A(x) dx$.

(ii) For cross sections of area $A(y)$ taken perpendicular to the y -axis, volume is $V = \int_c^d A(y) dy$.

27.) Solving Differential Equations Graphically

Slope Fields

At every point (x, y) a differential equation of the form $\frac{dy}{dx} = f(x, y)$ gives the slope of the member of the family of solutions that contains that point. A slope field is a graphical representation of this family of curves. At each point in the plane, a short segment is drawn whose slope is equal to the value of the derivative at that point. These segments are tangent to the solution curve's graph at the point.

The slope field allows you to sketch the graph of the solution curve even though you do not have its equation. This is done by starting at any point (usually the point given by the initial condition), and moving from one point to the next in the direction indicated by the segments of the slope field.

Some calculators have built-in operations for drawing slope fields; for calculators without this feature, there are programs available for drawing them.

28.) Solving Differential Equations by Separating the Variables

There are many techniques for solving differential equations. Any differential equation you may be asked to solve on the BC Calculus Exam can be solved by separating the variables (through multiplication or division only). Rewrite the equation as an equivalent equation with all the x and dx terms on one side and all the y and dy terms on the other. Antidifferentiate both sides to obtain an equation without dx or dy , but with one constant of integration ($+ C$). Use the initial condition to evaluate this constant.

29.) Law of Exponential Change

If a quantity y is changing according to the law of exponential change, its rate of change is modeled by the differential equation $\frac{dy}{dt} = ky$, where k is a constant. ($k > 0$ models growth; $k < 0$ models decay) You may separate the variables to solve for the function y ; however, it is known that the solution to the differential equation is $y = y_0 e^{kt}$, where y_0 is the initial amount.