① $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$ The function $a(n) = \frac{(\ln n)^2}{n}$ is positive for $n \geq 1$, it is continuous for $n \geq 1$, and for $n \geq 8$ is decreasing ($a'(n) < 0$ for $n > e^2$).

Therefore, we can use the Integral Test. $\int_1^{\infty} a(n)\,dn = \infty$, so, by the Integral Test, because this integral diverges, so does $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n}$.

② $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^5+7}}$ Use the Limit Comparison Test; for $a_n = \frac{1}{\sqrt{n^5+7}}$, $a_n > 0$ for $n \geq 1$; compare to $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5}} = \sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$.

By the p-series test, we know $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$ converges, $p = 5/2 > 1$. So the $\lim_{n \to \infty} \frac{a_n}{1/n^{5/2}} = 1$; a finite positive value; because $\sum \frac{1}{n^{5/2}}$ converges, so does $\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^5+7}} = \frac{1}{\sqrt{7}} + \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^5+7}}$, by the Limit Comparison Test.

③ $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{n}}}$ For $a(n) = \frac{1}{n^{\sqrt{n}}}$, $a(n)$ is positive, continuous, and decreasing (ck: $a'_n < 0$ for $n \geq 1$), so, use the Integral Test $\int^{\infty} a(n)\,dn \approx 1.056$, so, because this integral is convergent, so is $\sum_{n=1}^{\infty} a(n) = \sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{n}}}$.

④ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+7}{5^n} = \sum_{n=1}^{\infty}\left(\frac{2}{5}\right)^n + 7\sum_{n=1}^{\infty}\left(\frac{1}{5}\right)^n$; each of these are geometric series, the first has $r=\frac{2}{5}$, $\left|\frac{2}{5}\right|<1$; second has $r=\frac{1}{5}$, $\left|\frac{1}{5}\right|<1$, so each of the two series converges & therefore, so does the sum, $\sum \frac{2^n+7}{5^n}$. $S = S_1 + 7\cdot S_2 = \frac{2/5}{1-2/5} + \frac{7(1/5)}{1-1/5} = \frac{2}{3}+\frac{7}{4}$

$= 29/12$

⑤ $\displaystyle\sum \frac{5n^2+9}{7n^3-2n+8}$ Use Limit Comparison Test, compare to $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ which we know diverges, by p-series test, $p=1$, $1 \not> 1$.

So $\displaystyle\lim_{n\to\infty} \frac{\frac{5n^2+9}{7n^3-2n+8}}{1/n} = \frac{5}{7}$. Because this limit is a positive finite value, with $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ divergent, we know $\displaystyle\sum \frac{5n^2+9}{7n^3-2n+8}$ also Diverges, by the Limit Comparison Test.

⑥ $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}}{(2n)!}$ Let $a_n = \frac{2^{2n}}{(2n)!}$; then $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{4(2n)!}{(2(n+1))!}\right|$

$\displaystyle= \lim_{n\to\infty}\left|\frac{4(2n)!}{(2n+2)\cdot(2n+1)(2n!)}\right| = \lim_{n\to\infty}\left|\frac{4}{(2n+2)(2n+1)}\right| = 0 = L$

Because $L=0<1$, the series $\sum \frac{2^{2n}}{(2n)!}$ converges, by the Ratio Test.

(7) $\sum_{n=1}^{\infty} \frac{7}{3^n+2}$; For $a_n = \frac{7}{3^n+2}$, $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{3}$, $\frac{1}{3} = L$, $\frac{1}{3} < 1$, so by the Ratio Test the series Converges.

(8) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1} = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{2n+1}$ Use the Alternating Series Test with $b_n = \frac{1}{2n+1}$, we have $\lim_{n\to\infty} b_n = 0$ and b_n is decreasing as $n \to \infty$ (denom. grows larger) This means we've met criteria for the A.S.T. & this series Converges.

(9) $\sum_{n=1}^{\infty} \frac{3^n}{n \cdot 2^n}$; $\lim_{n\to\infty}\left|\frac{3^{n+1}/(n+1)\cdot 2^{n+1}}{3^n/n\cdot 2^n}\right| = \frac{3}{2} = L$, & $L > 1$, so this series Diverges by the Ratio Test.

(10) $\sum_{n=1}^{\infty} \frac{n}{n+2}$; Look at $\lim_{n\to\infty} \frac{n}{n+2} = 1$; Because this limit is <u>not</u> 0, we know this series Diverges by the Divergence Test.