Math 145, Fall 2008
H. Jordon

Solutions to Section 2.8, #55

Section 2.8:

55. Show that (a) the derivative of an even function is an odd function and that (b) the derivative of an odd function is an even function. Recall that a function is **even** if $f(x) = f(-x)$ for all x and a function is **odd** if $f(x) = -f(-x)$ for all x.

First, note that if $\lim_{t\to 0} g(t)$ exists, then $\lim_{t\to 0} g(t) = \lim_{t\to 0} g(-t)$ since $\lim_{t\to 0^+} g(-t) = \lim_{t\to 0^-} g(t) = \lim_{t\to 0} g(t)$ and $\lim_{t\to 0^-} g(-t) = \lim_{t\to 0^+} g(t) = \lim_{t\to 0} g(t)$ so that $\lim_{t\to 0} g(-t) = \lim_{t\to 0} g(t)$.

(a) Let f be an even function. We wish to show f' is an odd function, i.e., $-f'(x) = f'(-x)$ for all x. So,

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{f(-(x+h)) - f(-x)}{h} && \text{since } f \text{ is even} \\
&= \lim_{h\to 0} \frac{f(-x-h) - f(-x)}{h} \\
&= -\lim_{h\to 0} \frac{f(-x-h) - f(-x)}{-h} \\
&= -\lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} && \text{since } \lim_{t\to 0} g(t) = \lim_{t\to 0} g(-t) \\
&= -f'(-x).
\end{aligned}$$

(b) Let f be an odd function. We wish to show f' is an even function, i.e., $f'(-x) = f'(x)$. So,

$$\begin{aligned}
f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h\to 0} \frac{-f(x+h) + f(x)}{-h} \\
&= \lim_{h\to 0} \frac{f(-(x+h)) - f(-x)}{-h} && \text{since } f \text{ is odd} \\
&= \lim_{h\to 0} \frac{f(-x-h) - f(-x)}{-h} \\
&= \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} && \text{since } \lim_{t\to 0} g(t) = \lim_{t\to 0} g(-t) \\
&= f'(-x).
\end{aligned}$$