

Name: _____

Math 145, Fall 2002

H. Gavlas

Exam 3

100 points maximum. Calculators may be used on some questions. To receive full credit, you **must** show all work!

1. (15 pts.) Answer the following questions by circling **TRUE** or **FALSE**. BE CAREFUL in choosing your answer. No explanation necessary.

(a) **TRUE** or **FALSE** If $f'(c) = 0$, then f has a local minimum or local maximum at $x = c$.

(b) **TRUE** or **FALSE** $\frac{d}{dx}f(\ln x) = \frac{1}{f'(x)} \frac{1}{x}$

(c) **TRUE** or **FALSE** The local minimums/maximums of f' are the inflection points of f .

(d) **TRUE** or **FALSE** If $f''(c) = 0$, then f has a point of inflection at $(c, f(c))$.

(e) **TRUE** or **FALSE** There exists a function f such that $f(1) = -3$, $f(4) = 0$, and $f'(x) > 1$ for all x .

2. (12 pts.) Let $f(x) = \sqrt[3]{1+x}$.

(a) Find an equation of the tangent line to the graph of $y = f(x)$ at the point $(0, 1)$.

(b) Use your work in part (a) to find an approximate value for $\sqrt[3]{1.25}$. Give your answer with 6 significant digits. (**Note:** I do not want the approximation you get by entering $1.25^{(1/3)}$ on your calculator.)

3. (15 pts.) A balloon is being filled with water at a constant rate of 8 ml/s. Assume that as the balloon is being filled, it forms a sphere. At what rate is the radius of the balloon increasing when it is 64 ml full? (1 ml = 1 cm³)

4. (15 pts.) Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x -axis and lying on the parabola $y = 8 - x^2$.

5. (20 pts.) Let $y = e^{2x-x^2}$. In each of the following questions, use Calculus to find:

(a) the first derivative;

(b) the critical points;

(c) the local minimums and maximums and the intervals of increase or decrease;

(d) the second derivative; and

(e) the intervals of concavity and the inflections points.

6. (10 pts.) Differentiate the following functions. **Do not simplify your answers or use your calculator.**

(a) $y = \ln(x^3 e^x)$

(b) $y = x^{\sin x}$

7. (13 pts.) A particle moves on a vertical line so that its position at time t is $y = t^3 - 12t + 3$, $t \geq 0$.

(a) Find the velocity function.

(b) Use the velocity function to find the time intervals when the particle moving upward and when is it moving downward.

(c) Find the distance that the particle travels in the time interval $0 \leq t \leq 3$.