

Name: \_\_\_\_\_

Math 145, Fall 2002

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## Exam 2

There are 105 points possible with 100 points maximum. Calculators may **not** be used. To receive full credit, you **must** show all work!

1. (15 pts.) Answer the following questions by circling **TRUE** or **FALSE**. BE CAREFUL in choosing your answer. No explanation necessary.

(a) **TRUE** or **FALSE** If  $f'(a)$  exists, then  $f$  is continuous at  $x = a$ .

(b) **TRUE** or **FALSE**  $\frac{d}{dx}\pi^2 = 2\pi$

(c) **TRUE** or **FALSE** If  $f$  is a differentiable function, then  $\frac{d}{dx}f(e^{x^2}) = f'(e^{2x})e^{x^2}$ .

(d) **TRUE** or **FALSE** If a function  $f$  is concave up on  $(a, b)$ , then  $f'$  is decreasing on  $(a, b)$ .

(e) **TRUE** or **FALSE**  $\lim_{h \rightarrow 0} \frac{(2+h)^4 - 16}{h} = 32$

2. (8 pts.) Give an example of each of the following or state that no such example exists. No explanation necessary.

(a) The graph of a function  $f$  such that  $f'' < 0$  everywhere,  $f' > 0$  everywhere, and  $f < 0$  everywhere.

(b) The graph of a function  $f$  such that  $f'' > 0$  everywhere,  $f' > 0$  everywhere, and  $f > 0$  everywhere.

3. (15 pts.) Answer each of the following:

(a) Define the **derivative** of a function  $f$  at a **number**  $a$ .

(b) Give two interpretations of  $f'(a)$ .

(c) Find  $f'(a)$  **using the definition** if  $f(x) = \frac{4x}{x+1}$ .

4. (10 pts.) Find  $\frac{dy}{dx}$  if  $2x^2 + 6xy^2 - 7y^3 = 1$ .

5. (5 pts.) Find  $\lim_{t \rightarrow 0} \frac{t}{\tan t}$ .

6. (30 pts.) Differentiate the following functions. **Do not simplify your answers.**

(a)  $y = x \tan^{-1}(x + \pi)$

(b)  $y = e^{x \cos x}$

(c)  $y = \frac{x}{1 + \sin x}$

(d)  $w = \left( z + \frac{1}{z} + \frac{2}{z^2} \right)^3$

(e)  $s(t) = \frac{(t^2 + 1)^3}{(4t - 5)^5}$

7. (10 pts.) The temperature  $H$  in degrees Fahrenheit of a can of soda that has been put into a refrigerator to cool is given as a function of time  $t$  in hours, that is,  $H = f(t)$  for some function  $f$ .

(a) What is the meaning of the derivative  $f'(t)$ ? What are its units?

(b) What does the statement  $f'(2) = -4$  mean?

(c) Is  $f'(t)$  always negative or does it change sign?

8. (12 pts.) A table of values is given below for  $f$ ,  $g$ ,  $f'$ , and  $g'$ .

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

Find each of the following:

(a)  $(fg)'(2)$

(b)  $\left(\frac{f}{g}\right)'(3)$

(c)  $(f \circ g)'(1)$