Math 146
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Section 9.2: Euler's Method

In Section 9.2, we solve differential equations graphically with direction fields and numerically using **Euler's Method.** Euler's Method is based on two important facts: (1) the derivative gives the **slope** of the function at the point (x, y) and (2) the point-slope equation of a line. Euler's Method is an **iterative** process for approximating the value of the function at $x = a$, given some starting point $x = x_0$, the derivative $y' = \frac{dy}{dx}$, and a **step size** h, which describes how often the process must be updated.

(1) Let's start with the point-slope equation of a line: Given a point (x_0, y_0) and the slope m, what's the point-slope equation of the line through (x_0, y_0) with slope m? Solve your equation for y.

(2) Now, let's use Euler's method with step size $h = 0.2$ to approximate the solution to $y(1)$ if $y' = \frac{y}{x^2 + 1}$ and $y(0) = 2$.

(a) First, how many steps will it take to get from 0 to 1 if our step size is $h = 0.2$?

(b) In the first question, you should have found $y = y_0 + m(x - x_0)$. Since our step size is $h = 0.2$, what do you think $x - x_0$ will be in this process?

(c) Here is how Euler's Method works: Suppose we are given a step size h and our starting point $x = x_0$ and ending point $x = x_n$ (so it takes n steps to go from x_0 to x_n). We use the derivative to find the slope at (x_0, y_0), say

$$m_0 = y'(x_0, y_0).$$

Then

$$y_1 = y_0 + hm_0.$$

Using the fact that $y(0) = 2$ and $y' = \frac{y}{x^2 + 1}$ with step size $h = 0.2$, find y_1 and determine the next point (x_1, y_1).

(d) Now, we have a new point (x_1, y_1) and we use this point to find the next y-value by first finding the new slope $m_1 = y'(x_1, y_1)$ and then $y_2 = y_1 + m_1 \cdot h$. We continue until we find y_n which is our approximation to $y(x_n)$. Use your work in the previous questions and continue the process by filling in the table below to approximate $y(1)$. Use the fact that for $i \geq 1$,

$$y_i = y_{i-1} + hm_{i-1}.$$

Use approximate function values that are correct to four decimal places.

i	x_i	y_i	$m_i = y'(x_i, y_i)$
0			
1			
2			
3			
4			
5			

(3) Use Euler's method with step size $h = 0.1$ to approximate the solution to $y(1.4)$ if $y' = \dfrac{x}{2+y}$ and $y(1) = 1$. To answer this question, fill in the table below. Label each column and use approximate function values that are correct to four decimal places.

i			
0			
1			
2			
3			
4			