

Section 6.1: Area between Two Curves

You have already seen that if f is a **positive** function on $[a, b]$, then $\int_a^b f(x)dx$ gives the **area** under the curve $y = f(x)$ bounded by the x -axis and the lines $x = a$ and $x = b$. What if we have **two** functions bounding a region? That is what you are going to investigate.

In each of the problems below, do the following:

- Graph both equations on a common screen. Pick an appropriate viewing window.
- Draw a **typical rectangle** inside the region (i.e. imagine “filling” the region up with rectangles). Find the height of the rectangle and the length of its base.
- Set up and evaluate a **definite integral** that will give the area of the region bounded by the graphs of the equations. Do not use two definite integrals to find the area.

1. $y = x^2, y = \sqrt{x}$

2. $y + x^2 = 6, y + 2x = 3$

3. $2y^2 = x + 4$, $x = y^2$ (**Hint:** Think horizontally!)

4. **Conclusion:** Let f and g be functions with $f(x) \geq g(x)$ for all x in $[a, b]$. Then the area of the region bounded by the curves $y = f(x)$, $y = g(x)$, $x = a$, and $x = b$ is given by the following definite integral: