Math 146
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Section 6.3: Volume by Cylindrical Shells

We have already learned several methods for determining the volume of a solid: by disc, by washer, or by slicing a solid into layers. In this handout, you are going to learn another method, called the **method of cylindrical shells**.

1. Consider solid obtained by rotating the region bounded by the graphs of $y = 2x - x^2$ and the x-axis about the y-axis.

   (a) Draw the region. Draw a **typical rectangle** inside the region, perpendicular to the y-axis.

   (b) Imagine rotating the rectangle around the given axis. **Draw** the result of rotating the rectangle. What is the result of this rotation? Can you determine the volume of this washer easily? Why or why not?

   (c) Re-draw the region and a typical rectangle inside the region but with your rectangle **perpendicular to the x-axis**.

(d) Rotate the rectangle around the y-axis. Draw the result of this rotation. Notice that we now have a **cylindrical shell**, that is, a very thin "soup can" without a top or bottom. Imagine you cut the "soup can" vertically and open it up to obtain a rectangle with a little bit of thickness. What's the length of the rectangle? height? thickness? Find the volume of this "soup can".

(e) Use your work from the previous question to find the volume of the solid.

2. Using the method of cylindrical shells, find the volume of the solid obtained by rotating the region bounded by the curves of $y = x^2$ and $y = x + 2$ about the axis $x = 3$.