Name: _____

Math 146, Fall 2009
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Exam 1

There are 102 points possible, 100 maximum. Calculators may be used on any question unless otherwise specified. To receive full credit, you **must** show all work! Answers without any work will be given little if any credit.

1. (15 pts.) Answer the following questions by circling **TRUE** or **FALSE**. BE CAREFUL in choosing your answer. No explanation necessary.

   (a) **TRUE** or **FALSE** If f is a continuous function on the interval $[a, b]$, then $\displaystyle\int_a^b xf(x)\ dx = x\int_a^b f(x)\ dx$.

   (b) **TRUE** or **FALSE** $\displaystyle\int \frac{f(x)}{f'(x)}dx = \ln|f(x)| + C$

   (c) **TRUE** or **FALSE** The definite integral $\displaystyle\int_0^{\pi/2} 2\pi\cos^2 x\ dx$ represents the volume of the solid obtained by rotating the region bounded by the curves $y = \cos x$ and $y = 0$ for $-\pi/2 \le x \le \pi/2$ about the x-axis.

   (d) **TRUE** or **FALSE** If f is continuous function on the interval $[0, 3]$, then $\displaystyle\int_0^3 f(x)\ dx = \int_0^3 f(3-x)\ dx$.

   (e) **TRUE** or **FALSE** If f is continuous function on the interval $[a, b]$, then there exists a real number c in $[a, b]$ such that $\displaystyle f(c) = \frac{1}{b-a}\int_a^b f(x)\ dx$.

2. (10 pts.) A force of 30 N is required to maintain a spring stretched from its natural length of 12 cm to a length of 15 cm.

   (a) Using Hook's Law, find the force function $f(x)$. What does x represent in this situation and what are its units?

   (b) Set up and evaluate a definite integral giving work done in stretching the spring from 12 cm to 20 cm. Clearly delineate the "force" and the "distance" in your expression.

3. (10 pts) Find the average value of the function $f(t) = t\cos(t^2)$ on the interval $[0, \sqrt{\pi/2}]$ **BY HAND**. This mean you may only use your calculator AS A CHECK, i.e., do not use your calculator for the integration.

4. (15 pts.) A tank, in the shape of a cylinder with height and radius 4 m, is filled with water to a depth of 3 m. All the water is to be pumped out of the tank, over the top. Useful fact: The density of water is 1000 kg/m^3.

   (a) Draw a picture to represent this situation. Draw a typical "slice".

   (b) For a typical "slice", find the work to move the "slice" to the top of the tank. Clearly delineate the "force" and the "distance" in your expression.

   (c) Set up and evaluate a definite integral giving the work required to empty the tank by pumping the water to the top of the tank.

5. (25 pts.) Let R be the region bounded by the graphs of $y = x$ and $y = 2 - x^2$.

(a) Sketch the region R. Set up and evaluate a definite integral that represents the area of R.

In parts (b) and (c) below, do the following:

i. Draw the region R and the line of rotation. Draw a typical rectangle inside the region. Label your picture appropriately for this situation, i.e., label the necessary quantities (height, radius, width, etc.) for obtaining the volume of the object obtained by rotating the rectangle about the given axis.

ii. Using your labels from the previous question, find the volume of the disc, washer, or shell obtained by rotating the typical rectangle.

iii. Set up and evaluate a definite integral giving the volume of the solid obtained by rotating the region about the given axis.

(b) Let S be the solid obtained by revolving R about the line $y = 3$.

(c) Let T be the solid obtained by revolving R about the line $x = 2$.

6. (15 pts.) A solid S has as its base the region in the xy-plane bounded by the graphs of $y = x^2$ and $y = 2$. Cross-sections perpendicular to the y-axis are rectangles whose heights are twice that of the side in the xy-plane.

   (a) Draw the base of the solid S and a typical cross-section. Appropriately label the typical cross-section.

   (b) Using your labels from the previous question, find the volume of the cross-section.

   (c) Set up and evaluate a definite integral giving the volume of the solid.

   (d) Suppose the cross-sections, which are still rectangles whose height are twice that of the side in the xy-plane, are now perpendicular to the x-axis. Set up and evaluate a definite integral giving the volume of the solid.

7. (12 pts) Evaluate each of the following integrals **BY HAND**. This means that you may only use your calculator **AS A CHECK**.

   (a) $\displaystyle\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$

   (b) $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx$