

Exam 3

105 points possible, 100 points maximum. To receive full credit, you **must** show all work! **Calculators may be used on all questions unless otherwise specified.**

1. (10 pts.) The length of time spent waiting in line at Watterson Food Court can be modeled by an exponential density function with mean 6 minutes.

(a) What is the probability that you will wait in line no more than 3 minutes?

(b) What is the probability that you will wait in line more than 3 minutes?

2. (12 pts.) Use Euler's method with step size $h = 0.2$ to approximate the solution to $y(1)$ if $y' = \frac{y}{x^2 + 1}$ and $y(0) = 2$. To answer this question, fill in the table below. Label each column and use approximate function values that are correct to four decimal places.

n			
0			
1			
2			
3			
4			
5			

3. (15 pts.) One model for the spread of an epidemic is that the rate of spread is jointly proportional to the number of infected people and the number of uninfected people. Suppose that an outbreak of a previously unknown type of influenza occurred in ISU's Watterson Towers. Due to the contagious nature of the disease, Watterson was quarantined and the disease was allowed to run its course. At the beginning of the outbreak, 100 people were infected. After two days, there were 1626 people infected. Answer the questions below, assuming that there were 6000 people living in Watterson Towers at the time of the outbreak.

(a) Let y denote the number of infected people and let t denote the number of days since the epidemic started. What differential equation and initial condition does y satisfy?

(b) Solve your differential equation.

(c) How many people will have been infected 7 days after the epidemic started?

4. (15 pts.) A tank contains 100 L of pure water. Brine that contains 0.05 kg of salt per liter of water enters the tank at a rate of 10 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. Let $y(t)$ represent the amount of salt in the tank after t minutes.

(a) Determine a differential equation that represents rate of change in the amount of salt in the tank.

(b) Solve your differential equation to find $y(t)$.

(c) How much salt is present in the tank after 30 minutes?

(d) How much salt is present after a long time?

5. (16 pts.) The end of a water trough is in the shape of an isosceles trapezoid 2 m wide at the bottom, 4 m wide at the top, and 1 m high. The trough is full of water.

(a) Draw a picture of this situation.

(b) Divide the end of the trough into sections. Imagine a arbitrary section. Label your picture in (a) appropriately. Determine the force exerted by the water on this section. Clearly delineate the “area”, “depth”, and “density” in your expression. (Recall that the density of water is 1000 kg/m^3 .)

(c) Use your work in the previous question to set up and evaluate a definite integral which will give the hydrostatic force exerted on one end of the trough.

6. (12 pts.) Find the solution to the differential equation $\frac{dy}{dx} + 2xy = y$ that satisfies the initial condition $y(0) = 5$.

7. (15 pts.) Consider the differential equation $x^2y' - y = 2x^3e^{-1/x}$.

(a) Is this differential equation a first-order linear equation? If so, what is $P(x)$? $Q(x)$?

(b) Find $I(x)$, the integrating factor.

(c) Find $\int I(x)Q(x) dx$.

(d) Using your work from the previous question, find y as a function of x .

8. (10 pts.) Find the **exact** length of the curve $y = \ln(\sec x)$ for $0 \leq x \leq \frac{\pi}{4}$. **Do not use your calculator on this question.**