Name: ______________________
Math 146, Fall 2009
H. Jordon

Exam 4

To receive full credit, you **must** show all work! 100 points possible.

1. (14 pts.) Find a power series representation for $f(x) = \ln(1 + 2x)$ and determine the radius of convergence.

2. (12 pts.) Determine, with justification, whether the following sequences are convergent or divergent. For those sequences that are convergent, find the exact value of the limit.

   (a) $\{\ln n - \ln(3n + 1)\}$

   (b) $a_n = \dfrac{2 + n^2}{1 + 2n^2}$ for $n \geq 1$

3. (14 pts.) Find the radius of convergence and interval of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n x^n}{n^2}$.

4. (10 pts.) Determine whether the series is convergent or divergent.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$

5. (16 pts.) Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

6. (12 pts.) For values of x does the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n}$ converge? Find the sum of the series for those values of x.

7. (10 pts.) Give an example of each of the following or state that no such example exists, with justification.

(a) A series $\sum_{n=1}^{\infty} a_n$ such that $\sum_{n=1}^{\infty} a_n$ is convergent yet $\sum_{n=1}^{\infty} |a_n|$ is divergent.

(b) Series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ such that $\sum_{n=1}^{\infty} (a_n + b_n)$ converges yet both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge.

8. (12 pts.) Determine, with justification, whether the following series are convergent or divergent. For those series that are convergent, find the sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1+3^n}{4^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{2n^2+n}$