Math 146
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Section 8.3: Hydrostatic Force and Pressure

Hydrostatic Force means the force exerted by a liquid. For example, imagine you are a deep-sea diver. As you dive deeper, the water pressure increases because the weight of the water above you increases. In physics, the force exerted by a fluid on an object at a **depth** d below the surface is the **area** A of exposure of the object times the depth d times the **density** ρ of the fluid, i.e.,

$$F = A \times d \times \rho.$$

Force F is measured in **pounds** lb or **newtons** N. It is important to note that **pounds** lb account for the force due to gravity **and** mass while kilograms only accounts for mass. Thus, to determine the force F in **newtons** N, we use **mass** m kg times acceleration due to gravity $g = 9.8$ m/s^2, that is N = kg $\times$ m/s^2. The **area** will be either in square meters m^2 or square feet ft^2; the **depth** will be measured in meters or feet; and the **density** will be in kg/m^3 or lb/ft^3. Let the unit helps you!

1. A rectangular fish tank is 2 ft wide, 4 ft long, and 3 ft high and is filled with water. Calculate the force on the bottom of the tank due to hydrostatic pressure. Remember that the density of water is 62.5 lb/ft^3.

   In the previous problem, the **depth** was **constant**, but what if it isn't? What if the object is submerged vertically in the liquid? That is what we are going to consider next.

2. Suppose we have the same fish tank as in the previous problem. In the following steps, we are going to calculate the hydrostatic force on the "front" plate of glass, say the side that is 3 ft $\times$ 4 ft.

   (a) **Draw** a picture of the fish tank (fish not necessary). Consider the front of the tank in **horizontal** layers. What's the shape of an arbitrary "layer"? Is each arbitrary layer at the **same** depth? Will some layers have more pressure due to hydrostatic force acting on them than others? Why or why not?

   (b) Determine the dimensions of an arbitrary "layer". Determine the area of the glass in the layer (units are important here).

(c) Use your work in the previous question to find the **force** on an arbitrary layer of the front of the tank.

(d) Use your answers to the previous questions to set up and evaluate a definite integral which will give the hydrostatic force on the front of the fish tank. In your definite integral, identify each of the following: area, depth, density.

3. A cylindrical oil storage tank is 4 m in diameter and 5 m long, and is lying on its side. The tank is half-full of oil weighing 60 kg/m^3. We want to find the force exerted by the oil on one end of the tank.

   (a) Draw a picture of this situation. You may want to impose a coordinate system on your picture.

   (b) Divide the end of the tank into sections. Imagine a arbitrary section. Is each section at the same depth in the oil? Will some sections have more pressure due to the force of the oil acting on them than others? Explain.

   (c) Determine the area of an arbitrary section of the end of the tank. Use this to determine the force exerted by the oil on this section. Watch your units!

   (d) Use your work in the previous question to set up and evaluate a definite integral which will give the force exerted by the oil on the end of the tank. In your definite integral, identify each of the following: area, depth, density.