Section 7.8: Improper Integrals, Type I

Definition 1 An *improper integral* is a definite integral of the form $\int_a^\infty f(x)\ dx$.
If $\int_a^t f(x)\ dx$ exists for every $t \geq a$, then

$$\int_a^\infty f(x)\ dx = \lim_{t\to\infty} \int_a^t f(x)\ dx.$$

If this limit exists, we say the improper integral *converges;* otherwise the improper integral *diverges.*

Another example of an improper integral is $\int_{-\infty}^a f(x)\ dx$, which is evaluated similarly. To evaluate $\int_{-\infty}^\infty f(x)\ dx$, we consider **two** improper integrals $\int_0^\infty f(x)\ dx$ and $\int_{-\infty}^0 f(x)\ dx$, and $\int_{-\infty}^\infty f(x)\ dx$ converges to the sum of these two improper integrals, provided they both converge.

(1) Consider $\displaystyle\int_1^\infty \frac{1}{x} dx$.

(a) Is this an improper integral? Why or why not?

(b) Using the definition above, set up the appropriate limit that would need to be evaluated to determine whether this improper integral converges.

(c) Notice that in your answer to the previous question, you have both a limit and a definite integral to evaluate. Which must you determine first? Do so below.

(d) Now take the limit. Does this improper integral converge or diverge?

(e) What does that tell you about the area under the curve of $y = 1/x$ from $[1, \infty)$?

(2) Consider $\int_1^\infty \frac{1}{x^2} dx$.

(a) Using the definition above, set up the appropriate limit that would need to be evaluated to determine whether this improper integral converges.

(b) Again, you have a definite integral and a limit to evaluate. Evaluate the definite integral below first.

(c) Now take the limit. Does this improper integral converge or diverge?

(d) What does that tell you about the area under the curve of $y = 1/x^2$ from $[1, \infty)$?

(3) Determine whether $\int_1^\infty \frac{1}{\sqrt{x}} dx$ converges or diverges. If this improper integral converges, find it's value.

(4) Determine whether $\int_1^\infty \frac{1}{x^3} dx$ converges or diverges. If this improper integral converges, find it's value.

(5) Based on your work above, make a guess about the values of p for which the improper integral $\int_1^\infty \frac{1}{x^p} dx \ldots$

(a) converges

(b) diverges