Math 146
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Section 7.1: Integration by Parts

Integration by parts is an integration technique based on the product rule. Suppose we have functions u and v of x. Then, we know

$$\frac{d}{dx}\, uv = u\, \frac{dv}{dx} + v\, \frac{du}{dx}$$

or

$$(uv)' = uv' + vu'$$

or

$$uv' = (uv)' - vu'.$$

Thus,

$$\int uv'\ dx = \int (uv)'\ dx - \int vu'\ dx,$$

or, since integration and differentiation are inverse operations,

$$\int uv'\ dx = uv - \int vu'\ dx,$$

or

$$\int u\ dv = uv - \int v\ du.$$

This is the technique of integration by parts; we think of the integral as $\int u\ dv$ and use this information to find du and v. Note that when we apply integration by parts, we have an integral in our answer so there is still more work to do! However, it should be the case that the expression $\int v\ du$ is no worse than $\int u\ dv$. The key to integration by parts is to split the integrand into u and dv wisely. Note that if we are integrating $\int f(x)\ dx$ using integration by parts, then $f(x)\ dx = u\ dv$ and dx is always a part of dv. A mnemonic for choosing u is LIATE which stands for:

- L – logarithms
- I – inverse trig functions
- A – algebraic functions (like polynomials)
- T – trig functions
- E – exponential functions

Once we choose u and dv, we differentiate u to find du and integrate dv to find v.

Evaluate the following integrals using integration by parts.

1. $\int xe^{2x}\ dx = uv - \int v\ du =$ ______________________________ $=$

$u =$ ___________ $dv =$ ___________

$du =$ ___________ $v = \int dv =$ ___________

2. $\int x\sec^2 x\ dx = uv - \int v\ du =$ ______________________________ $=$

$u =$ ___________ $dv =$ ___________

$du =$ ___________ $v = \int dv =$ ___________

3. $\int \ln x\ dx = uv - \int v\ du =$ ______________________________ $=$

$u =$ ___________ $dv =$ ___________

$du =$ ___________ $v = \int dv =$ ___________

4. $\int \cos xe^{x}\ dx = uv - \int v\ du =$ ______________________________ $=$

$u =$ ___________ $dv =$ ___________

$du =$ ___________ $v = \int dv =$ ___________

5. $\int_0^1 x^2e^{3x}\ dx = uv - \int v\ du =$ ______________________________ $=$

$u =$ ___________ $dv =$ ___________

$du =$ ___________ $v = \int dv =$ ___________