**Math 146**
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# 11.8: Power Series

We are now going to use infinite series to represent **functions**; for example the series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, that is to say the series $\sum_{n=0}^{\infty} x^n$ represents the function $\frac{1}{1-x}$ for certain values of $x$ . Such a series is called a **power series**.

**Definition 1** The series

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots + c_n(x-a)^n + \cdots$$

is called a *power series.*

So, if we are thinking of our power series as a function $x$, then what we want to know is for which **values** of $x$ does the series **converge**? Note that $x = a$ always gives convergence (because then the series is $\sum_{n=0}^{\infty} c_n 0^n$). In fact, it turns out that there are only three different things that can happen.

**Theorem 2** *If* $\sum_{n=0}^{\infty} c_n(x-a)^n$ *is a power series, then precisely one of the following is true.*

*(1) The series converges only if* $x = a$.

*(2) There is a positive number* $R$ *such that the series is absolutely convergent if* $|x-a| < R$ *and divergent if* $|x-a| > R$.

*(3) The series is absolutely convergent for all* $x$.

In (2) in Theorem 2, we do not know what happens when $x - a = R$ or $x - a = -R$. These values, $x = a + R$ and $x = a - R$ have to be investigated separately.

**Definition 3** The *radius of convergence* in (1) is 0, in (2) is R, and in (3) is $\infty$. The *interval of convergence* is $\{a\}$ in (1), is $(-\infty, \infty)$ in (3), and is $(a-R, a+R), [a-R, a+R), (a-R, a+R]$ or $[a-R, a+R]$ in (2).

In Section 11.8, we are concerned about for which **values** of $x$ the power series is convergent; in Sections 11.9 and 11.10, we will be concerned about which **functions** the power series represent.

(1) Consider the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$.

(a) Determine $|a_{n+1}/a_n|$.

(b) Determine $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

(c) Applying the Ratio Test to your answer in the previous question, for which values of $x$ would we get convergence?

(d) What is radius of convergence for this series? What is the interval of convergence? Note that if the radius of convergence is $0 < R < \infty$, then to determine the **interval** of convergence, you must determine whether the series converges or diverges for $x = a + R$ and $x = a - R$.

(2) Consider the series $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$.

(a) Determine $|a_{n+1}/a_n|$.

(b) Determine $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

(c) Applying the Ratio Test to your answer in the previous question, for which values of $x$ would we get convergence?

(d) What is radius of convergence for this series? What is the interval of convergence? Note that if the radius of convergence is $0 < R < \infty$, then to determine the **interval** of convergence, you must determine whether the series converges or diverges for $x = a + R$ and $x = a - R$.

(3) Consider the series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

(a) Determine $|a_{n+1}/a_n|$.

(b) Determine $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

(c) Applying the Ratio Test to your answer in the previous question, for which values of $x$ would we get convergence?

(d) What is radius of convergence for this series? What is the interval of convergence? Note that if the radius of convergence is $0 < R < \infty$, then to determine the **interval** of convergence, you must determine whether the series converges or diverges for $x = a + R$ and $x = a - R$.

(4) Consider the series $\sum_{n=1}^{\infty} n!(2x-1)^n$.

(a) Determine $|a_{n+1}/a_n|$.

(b) Determine $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

(c) Applying the Ratio Test to your answer in the previous question, for which values of $x$ would we get convergence?

(d) What is radius of convergence for this series? What is the interval of convergence? Note that if the radius of convergence is $0 < R < \infty$, then to determine the **interval** of convergence, you must determine whether the series converges or diverges for $x = a + R$ and $x = a - R$.