Math 146
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Section 6.2: Volume by Slicing

We have seen how Calculus can be used to find the volume of a solid S obtained by revolving a region R about a given line. The process in general was to (1) divide the region into rectangles, (2) rotate the rectangle about the given line, (3) determine the volume of the object resulting from the rotation of the rectangle (either a "disc" or a "washer"), and finally (4) setup and evaluate a definite integral to find the volume of the solid. In determining (3), the volume of the object resulting from rotating the rectangle about the given line, we found the cross-sectional area, either $A(x)$ or $A(y)$, and then multiplied by the "thickness", either dx or dy, depending whether we divided the the region R into rectangles vertically (dx) or horizontally (dy).

Now we are going to find the volume of a given object, for example a sphere, by (1) "slicing" the object into sections, either horizontally or vertically, (2) finding the cross-sectional area $A(x)$ or $A(y)$ of the "slice", (3) determining the volume of the "slice" which will be either $A(x)dx$ (slicing vertically) or $A(y)dy$ (slicing horizontally), and finally (4) setting up and evaluating a definite integral to find the volume of the given object.

In each of the problems below, do the following:

- Draw the solid or at least the base of the solid. Impose a co-ordinate system on your picture.
- Draw a **typical slice. Find** the cross-sectional area of the slice. This will require **labelling** the slice with the appropriate values: for a square, we need the length of a side; for a triangle, we need the height and the base, etc. It's ok to have the cross-sectional area in terms of both x and y (and even in terms of other symbols that you may have introduced at this point).
- Determine if you are integrating with respect to x or y. Find the cross-sectional area $A(x)$ or $A(y)$ based on whichever variable you are integrating with respect to. To find the cross-sectional area as a function either x or y, you will probably need to determine a relationship between x and y, i.e., are x and y on a circle? on a line? etc.
- Set up and evaluate a **definite integral** that will give the volume of the solid.

1. Find the volume of a sphere of radius r.

2. The base of a solid S is the region in the xy-plane bounded by the graphs of $y = 4$ and $y = x^2$. Find the volume of S if parallel cross-sections perpendicular to the x-axis are squares

3. Find the volume of a right circular cone with height h and base radius r.

4. The base of a solid S is the region in the xy-plane bounded by the graphs of $y^2 = 4x$ and $x = 4$. Find the volume of S if parallel cross-sections perpendicular to the y-axis are semicircles with diameter in the xy-plane.