Math 146
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Section 5.5: The Substitution Rule

The substitution rule is an integration technique based on the chain rule. Suppose we want to integrate

$$\int f(x)f'(x)dx$$

where $f(x)$ is some function in x. Then, making the substitution $\boxed{u = f(x),}$ we have $\boxed{du = f'(x)dx}$ so that

$$\int f(x)f'(x)dx = \int u\ du.$$

Since $\int udu$ is easy to integrate, we can solve the problem! That is, $\boxed{\int u\ du = u^2/2 + C}$ so that

$$\int f(x)f'(x)dx = [f(x)]^2/2 + C.$$

To apply the substitution rule, we need to make a good choice for u. Really, the only way to get good at picking u is to PRACTICE! You will get better at it the more you practice. Here are some guidelines:

- Don't choose $u = x$. (Why not?)
- Choose u so that it's derivative, du, is 'hanging around'.
- The choice of u should make the integration <u>easier</u>, not harder.
- When making the substitution, <u>everything</u> in $f(x)dx$ must be accounted for.
- After making the substitution, you should be able to <u>integrate</u> with respect to u relatively easily. Of course, this means you have to know the basic integration rules.
- Be careful of choosing too much for u! Remember to leave something to integrate.

Evaluate the following integrals using the substitution rule.

1. $\int (2x^3 + 1)^7 x^2\ dx =$ _____ $=$

   $u =$ _____

   $du =$ _____ $\Rightarrow$ _____ $= x^2\ dx$

2. $\int \sin(5x)\ dx =$ _____ $=$

   $u =$ _____

   $du =$ _____ $\Rightarrow$ _____ $= dx$

3. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx =$ _____ $=$

$u =$ _____

$du =$ _____ $\Rightarrow$ _____ $= \frac{1}{\sqrt{x}}\,dx$

4. $\int \frac{e^{3/x}}{x^2}\,dx =$ _____ $=$

$u =$ _____

$du =$ _____ $\Rightarrow$ _____ $= \frac{1}{x^2}\,dx$

5. $\int e^x \cos(e^x)\,dx =$ _____ $=$

$u =$ _____

$du =$ _____

6. $\int_0^1 x\sqrt{2x+1}\,dx =$ _____ $=$

$u =$ _____ $\Rightarrow$ $x =$ _____

$du =$ _____

7. $\int_0^1 \frac{x}{3x^2-5}\,dx =$ _____ $=$

$u =$ _____

$du =$ _____