Section 7.2: Integrating Combinations of Trigonometric Functions

Given $\int \cos^m x \sin^n x \, dx$, here are some guidelines for evaluating:

Step 1: If m is odd, use the substitution $du = \cos x \, dx$ (so what is u in this case?) and convert remaining factors to sines using $\cos^2 x + \sin^2 x = 1$.

Step 2: If n is odd, use the substitution $du = -\sin x \, dx$ (so what is u in this case?) and convert remaining factors to cosines using $\cos^2 x + \sin^2 x = 1$.

Step 3: If m and n are both even, use the identities $\sin^2 w = \frac{1-\cos(2w)}{2}$ and $\cos^2 w = \frac{1+\cos(2w)}{2}$ to convert to odd powers of cosine.

Evaluate the following integrals.

1. $\int \sin^2 x \, dx$

2. $\int_0^{\pi/2} \cos^3 x \sin^2 x \, dx$

3. $\int \sin^2(2x) \cos^2(2x) \, dx$

4. $\int \sin^3 x \cos^2 x \, dx$

Notice that the guidelines on the previous page handle all possible cases for integrating $\int \cos^m x \sin^n x \, dx$. We now wish to consider $\int \tan^m x \sec^n x \, dx$.

Given $\int \tan^m x \sec^n x \, dx$, here are some guidelines for evaluating:

Step 1: If n is even, use the substitution $du = \sec^2 x \, dx$ (so what is u in this case?) and convert remaining factors to tangents using $\sec^2 x = 1 + \tan^2 x$.

Step 2: If m is odd and $n \geq 1$, use the substitution $du = \sec x \tan x \, dx$ (so what is u in this case?) and convert remaining factors to secants using $\tan^2 x = \sec^2 x - 1$.

Notice this does not handle all possible cases of m and n for $\int \tan^m x \sec^n x \, dx$. For the other cases, the guidelines are not as clear cut and may take a combination of approaches. Two other facts that will help are:

- $\int \tan x \, dx = \ln|\sec x| + C$ (we saw this in class)
- $\int \sec x \, dx = \ln|\sec x + \tan x| + C$ (you can verify this by differentiating $\ln|\sec x + \tan x|$)

Evaluate the following integrals.

1. $\int \sec^4 x \, dx$

2. $\int_0^{\pi/3} \tan^3 x \sec^3 x dx$

3. $\int \sec^3 x \, dx$

4. $\int \tan^2 x \sec x \, dx$