

1. Give an example of a linear transformation from  $\mathbb{R}^4$  to  $\mathbb{R}^3$ .

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + 4x_4 \\ 5x_3 \\ 2x_2 - x_4 \end{bmatrix}$$

2. Give an example of a function that is NOT a linear transformation from  $\mathbb{R}^4$  to  $\mathbb{R}^3$ .

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 5x_1 + x_4 \\ 5x_3^3 \\ 5x_2 - x_4 \end{bmatrix}$$

3. True or False: If  $T: \mathbb{R}^n \rightarrow \mathbb{R}^m$  is a linear transformation, then there exists an  $m \times n$  matrix  $A$  such that  $T = T_A$ ; that is  $T(\mathbf{x}) = T_A(\mathbf{x}) = A\mathbf{x}$ .

True

4. What is the standard matrix? How is it defined for a linear transformation  $T$ ?

The standard matrix is a unique  $m \times n$  matrix whose columns are the images under  $T$  of the standard vectors for  $\mathbb{R}^n$ . It is defined as  $A = [T(\mathbf{e}_1) \ T(\mathbf{e}_2) \ \dots \ T(\mathbf{e}_n)]$ .

5. In Practice Problem 3 on pp. 175, what is the standard matrix for this linear transformation?

$$T(e_1) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad T(e_2) = \begin{bmatrix} 0 \\ -3 \end{bmatrix} \quad T(e_3) = \begin{bmatrix} -5 \\ 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & -5 \\ 0 & -3 & 4 \end{bmatrix}$$