Consider the basis $B = \left\{ b_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, b_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, b_3 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$ for R^3 given in Example 1.

1. Find scalars c_1, c_2, c_3 such that $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ satisfies

$u = c_1 b_1 + c_2 b_2 + c_3 b_3$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -3/2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & 2 \end{bmatrix} \text{ so } u = \frac{-3}{2} b_1 + \frac{1}{2} b_2 + 2b_3$$

2. Suppose v is vector such that the coordinates of v with respect to the basis B in Example 1 are $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_B$. What does this mean, that is, you can write v as a linear combination of the vectors in B with what scalars?

It means $v = 1b_1 + 2b_2 + 3b_3$.

3. Find the coordinates of v with respect to the standard basis for R^3.

$$v = 1b_1 + 2b_2 + 3b_3 = 1\begin{bmatrix}1\\1\\1\end{bmatrix} + 2\begin{bmatrix}1\\-1\\1\end{bmatrix} + 3\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}6\\5\\9\end{bmatrix}$$