1. If $\mathcal{B} = \{u_1, u_2, u_3\} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$ is a basis for R^3 and the basis $\mathcal{B}$-coordinates of v are $[1\ 2\ -1]^T$, then v is WHAT linear combination of the vectors in B?

$v = 1u_1 + 2u_2 - 1u_3$.

2. If $\mathcal{B}$ is the basis in Example 1 on p. 266, determine the coordinates of $v = [1\ 2\ -1]^T$ with respect to the standard basis.

$$v = 1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - 1 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

3. Suppose $v = [1\ 4\ 5]^T$. Since we did not specify a basis in respect to these coordinates, we assume the basis is the standard basis.

a) Find the basis $\mathcal{B}$-coordinates of v where $\mathcal{B}$ is the basis in Example 1 on p. 266.

$$v = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$\Rightarrow$

$$v = \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \frac{-7}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

b) Let B be the matrix whose columns are the vectors in the basis in Example 1 on p. 266. Compute $B^{-1}v$ and compare your answer to question (a).

$$B^{-1}v = \begin{bmatrix} 2 & 1/2 & -3/2 \\ 0 & -1/2 & 1/2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -7/2 \\ 1/2 \\ 4 \end{bmatrix}$$

The answers are equivalent.