1. What does the notation $\mathcal{M}_{m \times n}$ mean (see pp. 492)?

For any given positive integers m and n, $\mathcal{M}_{m \times n}$ denotes the set of all m x n matrices.

2. What does the notation $\mathcal{L}(R^n, R^m)$ mean (again see pp. 492)?

For any given positive integers m and n, $\mathcal{L}(R^n, R^m)$ denotes the set of all linear transformations from R^n to R^m.

3. What does the notation $\mathcal{P}_2$ mean (again see pp. 492)?

$\mathcal{P}_2$ denotes the set of all polynomials of at most degree 2, i.e.,
$\mathcal{P}_2 = \{a_2x^2 + a_1x + a_0 \mid a_0, a_1, a_2 \in R\}$

4. We have been studying the vector space R^n all semester. Give examples of three other vector spaces.

1. Function Spaces denoted by $\mathcal{F}(S) = \{ f \mid f: S \rightarrow S \text{ is a function}\}$ where S is a nonempty set
2. Matrix Spaces denoted by $\mathcal{M}_{m \times n}$
3. Linear Transformation spaces denoted by $\mathcal{L}(R^n, R^m)$
4. Set of polynomials of degree at most 2, $\mathcal{P}_2$

In what follows, let
* V = { (a, b) | a, b are real numbers}.
* For two "vectors" u = (a, b) and v = (c, d) of V, define "vector addition" as u + v = (a, b) + (c, d) = (a + b + 1, c + d).
* For a scalar k, define "scalar multiplication" as k*u = k*(a,b) = (a, kb).

5. Determine (2, 1) + (1, 1) and 2*(2, 1)?

(2, 1) + (1, 1) = (2 + 1 + 1, 1 + 1) = (4, 2)

2*(2, 1) = (2, 2*1) = (2, 2)

6. Does Axiom (1) in the Definition of a Vector Space hold, ie. is it true that for all "vectors" u and v in this set V, u + v = v + u (where vector addition is defined as above)?

No, for example (2, 1) + (4, 3) = (2 + 1 + 1, 4 + 3) = (4, 7) and
(4, 3) + (2, 1) = (4 + 3 + 1, 2 + 1) = (8, 3)
so (2, 1) + (4, 3) ≠ (4, 3) + (2, 1)

7. Does Axiom (2) in the Definition of a Vector Space hold?

No, for example
[(1, 2) + (3, 4)] + (5, 6) = (4, 7) + (5, 6) = (12, 11) and
(1, 2) + [(3, 4) + (5, 6)] = (1, 2) + (8, 11) = (4, 19) but
(12, 11) ≠ (4, 19).

In what follows, let
* V = { (a, b) | a, b are real numbers}.
* For two "vectors" u = (a, b) and v = (c, d) of V, define "vector addition" as u + v = (a, b) + (c, d) = (a + b + 1, c + d).
* For a scalar k, define "scalar multiplication" as k*u = k*(a,b) = (a, kb).

8. Does Axiom (6) in the Definition of a Vector Space hold?

Yes. Let u = (a, b) and c and d be scalars. From the definition of scalar multiplication above:
(cd)*u = (cd)*(a, b) = (a, (cd)b) = (a, c(db)) = c*(a, db) = c*[d*(a, b)] = c*(d*u).

9. Does Axiom (7) in the Definition of a Vector Space hold?

No, for example
3*[(1, 2) + (3, 4)] = 3*(4, 7) = (4, 21) and
3*(1, 2) + 3*(3, 4) = (1, 6) + (3, 12) = (8, 15)
(4, 21) ≠ (8, 15).

10. Do you think V together with this new definition of "vector addition" and "scalar multiplication" is a vector space? Why or why not?

No, because not all of the axioms hold.