1. Describe the procedure for finding the eigenvalues of an n x n matrix A.

1. Find the matrix A - tI, where t is a variable.
2. Calculate det(A - tI)
3. Solve det(A - tI) = 0

2. Now that you've found the eigenvalues, describe the procedure for finding the corresponding eigenvectors.

-after finding the eigenvalues, find $(A - \lambda I)$
-find the rref$(A - \lambda I)$
-find the general solution from the rref
-those vectors will be the eigenvectors

3. How do you decide if an n x n matrix A is diagonalizable?

An nxn matrix A is diagonalizable iff $A = PDP^{-1}$ where D is a diagonal matrix, and P is an inveritble matrix.

Matrix A is diagonalizable if it has n linearly independent eigenvectors.

4. Give an example of two vector spaces of dimension 4, neither of which is R^4.

$\mathcal{P}_3$ and $\mathcal{L}(R^2, R^2)$

$\mathcal{P}_3$, $\mathcal{M}_{2\,x\,2}$

$\mathcal{M}_{2x2}$, $\mathcal{L}(R^2 , R^2)$

5. For the two vector spaces you gave in Question 4, define an isomorphism from one to the other.

Let T: $\mathcal{M}_{2x2} \rightarrow \mathcal{P}_3$ be defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + bx + cx^2 + dx^3$$

T: $\mathcal{P}_3 \rightarrow \mathcal{M}_{2x2}$ be defined by $T(f(x)) = \begin{bmatrix} f(1) & f'(1) \\ f''(1) & f'''(1) \end{bmatrix}$