1. What is a linear transformation?

A function T from R^n to R^m, written T : R^n --> R^m, is called a linear transformation if, for all vectors u and v in R^n and all scalars c, T(u +v) = T(u) + T(v) and T(cu) = cT(u).

2. Let T: R^n --> R^m be a linear transformation. What is the domain of T? What is the codomain of T?

The domain of T is R^n and the codomain is R^m.

3. Give an example of a linear transformation from R^3 to R^2.

Let T : R^3 --> R^2 be defined by T(u) = Au, where u is a 3 x 1 vector and A is 2 x 3 matrix.

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T[-2 = [-1
1 0]
3]

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T([x_1)= [$3x_1$- $4x_2$
x_2 $2x_1 + x_3$]
x_3]

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$T_A(\mathbf{x}) = A\mathbf{x}$

We need a A to be a 2 x 3 and **x** to be a 3 x 1

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_A\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

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$$f\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} x_1 + x_2 + x_3 \\ x_1^2 \end{bmatrix}$$

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$$T\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

4. Let A be an m x n matrix.
i) What does the notation T_A mean? How is this function defined?

The function $T_A : R^n \dashrightarrow R^m$ which is defined by $T_A(x) = Ax$ for all x in R^n where A is an m x n matrix and T_A is called the matrix transformation induced by A.

ii) What is the domain of T_A? What is the codomain of T_A?

The domain of T_A is R^n and the codomain is R^m.

iii) Is T_A a linear transformation? Explain.

Yes, $T_A(u + v) = A(u + v) = Au + Av = T_A(u) + T_A(v)$ and $T_A(cu) = A(cu) = c(Au) = cT_A(u)$ for all vectors u and v in R^n and scalars c.