Name: ______________________

Math 175, Spring 2008
H. Jordon

Exam 2

100 points possible. Throughout this exam, $\mathbb{R}$ denotes the set of real numbers.

1. (12 pts.) Answer each of the following by CIRCLING True or False. No explanation necessary.

   (a) **True** or **False**: If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation with standard matrix A, then T is one-to-one if and only if rank $A = n$.

   (b) **True** or **False**: A linear transformation is uniquely determined by the images of the standard vectors in its codomain.

   (c) **True** or **False**: If A and C are matrices such that $AC = [0]$, then $A = [0]$ or $C = [0]$, where $[0]$ denotes the zero matrix.

   (d) **True** or **False**: If A is invertible, then the columns of A^{-1} are linearly independent.

2. (9 pts.) Let A be an $m \times n$ matrix, with $m \neq n$, such that the only solution to $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$. What three other things (or facts) can you say about A?

3. (9 pts.) Let B be an $n \times n$ matrix such that the only solution to $B\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$. What three other things (or facts) can you say are true about the matrix B that are not true about the matrix A in Question 2?

4. (16 pts.) Give an example of each of the following or state that no such example exists.

(a) 2×2 distinct matrices A and B, neither of which is the I_2 nor the zero matrix, such that $AB = BA$.

(b) A function $f : \mathbb{R}^2 \to \mathbb{R}^2$ that is not a linear transformation.

(c) A 2×2 matrices A and B such that A and AB are invertible, yet B is not invertible.

(d) The 3×3 elementary matrix that corresponds to elementary row operation that subtracts 3 times the second row from the first row.

5. (10 pts.) Let A be an $m \times n$ matrix and let $\{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_k\}$ be a set of linearly independent vectors in $\mathbb{R}^m$. Suppose that $A\mathbf{v}_1 = \mathbf{b}_1$, $A\mathbf{v}_2 = \mathbf{b}_2$, …, $A\mathbf{v}_k = \mathbf{b}_k$. Prove that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is a linearly independent set.

6. (10 pts.) Determine, with explanation, if the following matrices are invertible.

(a) $\begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ -4 & -9 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -5 & -4 \\ 0 & 3 & 4 \\ -3 & 6 & 1 \end{bmatrix}$

7. (10 pts.) Let A and B be an $m \times n$ matrices and let C be a $n \times r$ matrix. Without using Theorem 2.2, prove that $(A + B)C = AC + BC$.

8. (14 pts.) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T\left(\begin{bmatrix} 2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T\left(\begin{bmatrix} -4 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 1 \end{bmatrix}$.

(a) Determine $T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$ and $T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$.

(b) Determine the standard matrix of T.

(c) Determine $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$ for any $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ in $\mathbb{R}^2$.

(d) Determine, with explanation, if T is onto.

(e) Determine, with explanation, if T is one-to-one.

9. (10 pts.) Consider the following set S of vectors:

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ r \end{bmatrix} \right\}$$

(a) Determine, if possible, value(s) of r which would make the set S linearly dependent.

(b) Determine, if possible, value(s) of r which would make the set S linearly independent.