Math 175
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Section 1.4

In Section 1.4, we use our row operations–multiply by a scalar, interchange two rows, and add a multiple of one row to another–to find the reduced row echelon form (rref) of a matrix. The process is called Gaussian-Elimination.

Consider the following augmented matrix:

$$\begin{bmatrix} 0 & -3 & -6 & 4 & 9 \\ -1 & -2 & -1 & 3 & 1 \\ -2 & -3 & 0 & 3 & -1 \\ 1 & 4 & 5 & -9 & -7 \end{bmatrix}$$

1. Since there is a nonzero entry in the first column, we do have a pivot in the $(1,1)$-position. Interchange rows, if necessary, to obtain a nonzero entry in the $(1,1)$-position. Write the new matrix below. Be sure to indicate which rows were interchanged by using the notation $R_i \leftrightarrow R_1$ next to Row 1.

2. If necessary, multiply Row 1 by a scalar c so that the $(1,1)$-entry $= 1$. Again, note this using the notation $cR_1 \to R_1$ next to Row 1.

3. Now use the first row to obtain 0s in the other positions in Column 1. Do this by adding the appropriate multiple of Row 1 to Row i. Use the notation $cR_1 + R_i \to R_i$ and note this next to Row i.

4. Next, we locate the second pivot position. Is there a nonzero entry in Column 2? If so and this nonzero entry is not in the first row, then we have pivot position in the $(2, 2)$-position. At this point you have a choice as to how your proceed. The goal is to zero out the column below the second pivot and scalar the pivot entry to 1. We can do these operations in either order. Do so below, indicating the operations used.

5. Next, we locate the third pivot. Is there a nonzero entry in column 3 that isn't in a pivot row? If so, then we have a pivot in Column 3. Otherwise, we check Column 4 for a nonzero entry in a nonpivot row etc. As before, once the pivot is located, zero out the entries below the pivot and scale the pivot entry to 1 (in either order).

6. We continue in this manner until all the pivot positions have been located and all entries below pivot positions are zeroed out. At this point, we have 3 pivot positions and a zero row. Thus, there are only 3 pivots possible. Now, we must zero out the entries above the pivots. Working right to left and indicating the operations used, add multiples of Row 3 to Rows 1 and 2 to obtain zeros in the column above the third pivot. Next, add a multiple of Row 2 to Row 1 to obtain a zero above the second pivot. Now, you have obtained the rref of the matrix! We will see that everyone obtains the **same** reduced row echelon form of the matrix but not necessarily in the same steps. Thus, the rref is **unique**.