Section 4.3: The Dimension of the Subspaces Associated with a Matrix

1. Consider the matrix $A = \begin{bmatrix} 1 & 0 & -1 & -3 & 1 & 4 \\ 2 & -1 & -1 & -8 & 3 & 9 \\ 1 & 1 & -2 & -1 & 0 & 3 \\ -1 & 3 & -2 & 9 & -4 & -7 \\ 0 & 1 & 1 & 2 & -1 & -3 \end{bmatrix}$.

   (a) What are the dimensions of A?

   (b) Find rref(A). What's the rank of A?

   (c) Without finding bases for the given spaces, determine dim Col A, dim Row A, and dim Null A. Also, describe where each subspace lives, i.e., $\mathbb{R}^k$ for the correct values of k.

   (d) Find a basis for Row A.

   (e) Find a basis for Null A.

(f) Determine whether $\left\{ \begin{bmatrix} -2 \\ -6 \\ 0 \\ -8 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \\ 0 \\ 12 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ -9 \\ -3 \\ 7 \\ 3 \end{bmatrix} \right\}$ is a basis for Col A.

(g) First, determine dim Null A^T. Then find a basis for Null A^T.

2. Show the set $\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 9 \\ -3 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 15 \\ 4 \end{bmatrix} \right\}$ is a basis for the subspace

$$V = \left\{ \begin{bmatrix} r - s + 3t \\ 2r - t \\ -r + 3s + 2t \\ -2r + s + t \end{bmatrix} \in \mathbb{R}^4 \mid r, s, t \in \mathbb{R} \right\}.$$