Math 260
Fall 2008
H. Jordon

Solutions to Section 9.2 & 9.3 Graded Problems

Section 9.2:

2. (5 pts.) Prove by mathematical induction that $4(2^n) + 3$ is a solution to the recurrence relation $s_n = 2s_{n-1} - 3$ for $n \geq 1$ with the initial condition $s_0 = 7$.

PROOF. Let $s_n = 2s_{n-1} - 3$ for $n \geq 1$ with the initial condition $s_0 = 7$. Let $P(n) : s_n = 4(2^n) + 3$.
Basis: $P(0) : s_0 = 4(2^0) + 3$ is clearly true.
Induction Hypothesis: Let $k \geq 0$ be an integer. Assume $P(k)$ is true, that is $s_k = 4(2^k) + 3$. We wish to show that $P(k+1)$ is true, that is, $s_{k+1} = 4(2^{k+1}) + 3$. So,

$$\begin{aligned} s_{k+1} &= 2s_k - 3 \text{ since } k \geq 0 \\ &= 2(4(2^k) + 3) - 3 \\ &= 4(2^{k+1}) + 6 - 3 \\ &= 4(2^{k+1}) + 3. \end{aligned}$$

■

In Exercises 11–24, use the method of iteration to find a formula expressing s_n as function of n for the given recurrence relation and initial condition.

23. $s_n = ns_{n-1} + 1$, $s_0 = 3$

Solution:

$$\begin{aligned} s_0 &= 3 \\ s_1 &= 1s_0 + 1 \\ s_2 &= 2s_1 + 1 \\ &= 2(1 \cdot s_0 + 1) + 1 \\ &= 2 \cdot 1 \cdot s_0 + 2 \cdot 1 + 1 \\ &= 2!s_0 + 2! + 1 \\ s_3 &= 3s_2 + 1 \\ &= 3(2!s_0 + 2! + 1) + 1 \\ &= 3!s_0 + 3! + 3 + 1 \\ s_4 &= 4s_3 + 1 \end{aligned}$$

$$
\begin{aligned}
&= 4(3!s_0 + 3! + 3 + 1) + 1 \\
&= 4!s_0 + 4! + 4 \cdot 3 + 4 + 1 \\
&= 4!s_0 + \frac{4!}{1!} + \frac{4!}{2!} + \frac{4!}{3!} + \frac{4!}{4!} \\
s_5 &= 5s_4 + 1 \\
&= 5(4!s_0 + 4! + 4 \cdot 3 + 4 + 1) + 1 \\
&= 5!s_0 + 5! + 5 \cdot 4 \cdot 3 + 5 \cdot 4 + 5 + 1 \\
&= 5! \cdot 3 + \frac{5!}{1!} + \frac{5!}{2!} + \frac{5!}{3!} + \frac{5!}{4!} + \frac{5!}{5!} \\
&\vdots \\
s_n &= n! \cdot 3 + \frac{n!}{1!} + \frac{n!}{2!} + \cdots + \frac{n!}{n!} \\
&= n! \cdot 2 + \frac{n!}{0!} + \frac{n!}{1!} + \frac{n!}{2!} + \cdots + \frac{n!}{n!}
\end{aligned}
$$

Thus $s_n = n! \cdot 2 + \frac{n!}{0!} + \frac{n!}{1!} + \frac{n!}{2!} + \cdots + \frac{n!}{n!}$ for $n \geq 0$.

Section 9.3:

In Exercises 1–24, find an explicit formula for s_n if $s_0, s_1, s_2, \ldots$ is a sequence satisfying the given recurrence relation and initial conditions.

20. (5 pts.) $s_n = -8s_{n-1} - 15s_{n-2}$**,** $s_0 = 2$**,** $s_1 = 2$

Solution: First, we solve $x^2 = -8x - 15$ or $x^2 + 8x + 15 = 0$. The roots of this equations are $r_1 = -3$ and $r_2 = -5$. Thus, $s_n = c_1(-3)^n + c_2(-5)^n$. Now, $s_0 = c_1(-3)^0 + c_2(-5)^0$ or $2 = c_1 + c_2$. Next, $s_1 = c_1(-3)^1 + c_2(-5)^1$ or $2 = -3c_1 - 5c_2$. So, $c_1 = 2 - c_2$. Thus, $2 = -3(2 - c_2) + -5c_2$ or $8 = -2c_2$ or $-4 = c_2$. Hence, $c_1 = 6$ so that $s_n = 6(-3)^n - 4(-5)^n$.