Name: ____________________

Math 260, Fall 2007

H. Jordon

Exam 4

100 points possible.

1. (16 pts.)

   (a) How many 8-bit strings begin with 110?

   (b) How many 8-bit strings end with 100?

   (c) How many 8-bit strings begin with 110 **and** end with 100?

   (d) How many 8-bit strings begin with 110 **or** end with 100?

2. (8 pts.) Let G be a bipartite graph with 22 vertices and partite sets V_1 and V_2 such that $|V_1| = 10$ and every vertex of V_1 has degree 5. If every vertex of V_2 has degree 4 or 6, how many vertices of degree 6 does G have?

3. (15 pts.) Answer each of the following by CIRCLING True or False. No explanation necessary.

   (a) **True** or **False**: Let G be a graph with an even number of vertices such that every vertex has the same degree k. Then G has a perfect matching.

   (b) **True** or **False**: In a network N, it is always possible to find the value of a maximum flow.

   (c) **True** or **False**: If G is a bipartite graph with partite sets V_1 and V_2 such that there is a subset X of V_1 with $|X| > |A(X)|$, then G does not have a matching saturating V_1.

   (d) **True** or **False**: In a network N, it is possible to find a flow f and a cut $\{S, T\}$ such that $\text{cap}(S, T) < \text{val } f$.

   (e) **True** or **False**: All complete graphs with $n \geq 2$ vertices have perfect matchings.

4. (12 pts.) Find an explicit formula for a_n if $a_1 = 3$ and $a_n = 3a_{n-1} - 8$ for $n \geq 2$.

5. (12 pts.) Let $a_n = 6a_{n-1} - 9a_{n-2}$ for $n \geq 2$ with $a_0 = 1$ and $a_1 = 9$. Using the theory developed in class for second-order homogeneous linear difference equations, find an explicit formula for a_n.

6. (15 pts.) A total of 6 freshman, 5 sophomores, and 4 juniors have volunteered to serve on a 4-person committee. How many such committees are possible if

   (a) any student may serve on the committee?

   (b) at least one freshman must serve on the committee?

   (c) at least one freshman, at least one sophomore, and at least one junior must serve on the committee?

7. (10 pts.) Let G be a graph with an even number of vertices and a hamiltonian path. Prove G has a perfect matching.

8. (12 pts.) Find a maximum flow in the network shown below. Prove that your flow is maximum by finding a minimum cut.