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Exercise for October 29, 2005

Let X be the random outcome of tossing a fair six-faced die, and $Y_1 = \min(X_1, X_2, X_3)$, Y_2 (where $Y_1 \leq Y_2 \leq Y_3$, and $Y_3 = \max(X_1, X_2, X_3)$) be the order statistics from a random sample X_1, X_2, X_3 from the distribution of X . Find $\Pr(Y_1 = 3)$.

- A. $\frac{36}{216}$ B. $\frac{37}{216}$ C. $\frac{38}{216}$ D. $\frac{39}{216}$ E. $\frac{40}{216}$

Solution.

In order for $Y_1 = 3$ we must have all three of the elements of the random sample fall into the set of outcomes $\{3, 4, 5, 6\}$ but not all of them fall into the set $\{4, 5, 6\}$. Therefore, the probability sought is

$$\left(\frac{2}{3}\right)^3 - \left(\frac{1}{2}\right)^3 = \frac{8}{27} - \frac{1}{8} = \frac{64}{216} - \frac{27}{216} = \frac{37}{216}.$$

Answer B.

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