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Author of the ASM Manual (the “Been There Done That!” manual) for Course P/1: free excerpts and a practice exam available online <http://www.studymanuals.com/exam1.htm>

Course P/1 seminar: <http://www.math.ilstu.edu/actuary/prepcourses.html>

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Dr. Ostaszewski's online exercise posted March 12, 2005

An urn contains 100 lottery tickets. There is one ticket that wins \$50, three tickets that win \$25, six tickets that win \$10, and fifteen tickets that win \$3. The remaining tickets win nothing. Two tickets are chosen at random, with each ticket having the same probability of being chosen. Let X be the amount won by the one of the two tickets that gives the smaller amount won (if both tickets win the same amount, then X is equal to that amount). Find the expected value of X .

- A. 0.1348 B. 0.0414 C. 0.2636 D. 0.7922 E. Does not exist

Solution.

Note that you cannot have $X = 50$, because there is only one \$50 ticket. Thus, the possible values of X are: 25, 10, 3, and 0. Furthermore, $X = 25$ when we choose one \$50 ticket and

one \$25 ticket, and there $\binom{1}{1} \cdot \binom{3}{1}$ ways to do that, or when we choose two \$25

tickets, and there are $\binom{3}{2}$ ways to do that. Since there are $\binom{100}{2}$ ways to choose 2 tickets out of 100,

$$\Pr(X = 25) = \frac{\binom{1}{1} \cdot \binom{3}{1} + \binom{3}{2}}{\binom{100}{2}} = \frac{1 \cdot 3 + 3}{4950} = \frac{6}{4950} = \frac{1}{825}.$$

Furthermore,

$$\Pr(X = 10) = \frac{\binom{4}{1} \cdot \binom{6}{1} + \binom{6}{2}}{\binom{100}{2}} = \frac{4 \cdot 6 + 15}{4950} = \frac{39}{4950} = \frac{13}{1650}$$

(one \$10 ticket and one higher amount ticket, or two \$10 tickets),

$$\Pr(X = 3) = \frac{\binom{10}{1} \cdot \binom{15}{1} + \binom{15}{2}}{\binom{100}{2}} = \frac{10 \cdot 15 + 105}{4950} = \frac{255}{4950} = \frac{17}{330},$$

(one \$3 ticket and one higher amount ticket, or two \$3 tickets),
and finally

$$\Pr(X = 0) = \frac{\binom{25}{1} \cdot \binom{75}{1} + \binom{75}{2}}{\binom{100}{2}} = \frac{25 \cdot 75 + 2775}{4950} = \frac{4650}{4950} = \frac{31}{33}$$

(for $X = 0$, one \$0 ticket and one other ticket, or two \$0 tickets).

Thus

$$\begin{aligned} E(X) &= 25 \cdot \frac{1}{825} + 10 \cdot \frac{13}{1650} + 3 \cdot \frac{17}{330} + 0 \cdot \frac{31}{33} = \\ &= \frac{1}{33} + \frac{13}{165} + \frac{17}{110} = \frac{10}{330} + \frac{26}{330} + \frac{51}{330} = \frac{87}{330} \approx 0.2636. \end{aligned}$$

Answer C.

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