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Dr. Ostaszewski's online exercise posted March 12, 2005

An urn contains 100 lottery tickets. There is one ticket that wins \$50, three tickets that win \$25, six tickets that win \$10, and fifteen tickets that win \$3. The remaining tickets win nothing. Two tickets are chosen at random, with each ticket having the same probability of being chosen. Let X be the amount won by the one of the two tickets that gives the smaller amount won (if both tickets win the same amount, then X is equal to that amount). Find the expected value of X.

A. 0.1348 B. 0.0414 C. 0.2636 D. 0.7922 E. Does not exist

Solution.

Note that you cannot have $X = 50$, because there is only one \$50 ticket. Thus, the possible values of X are: 25, 10, 3, and 0. Furthermore, $X = 25$ when we choose one \$50 ticket and one \$25 ticket, are and there $\binom{1}{1}\cdot\binom{3}{1}$ ways to do that, or when we choose two \$25 tickets, and there are $\binom{3}{2}$ ways to do that. Since there are $\binom{100}{2}$ ways to choose 2 tickets out of 100,

$$\Pr(X = 25) = \frac{\binom{1}{1}\cdot\binom{3}{1}+\binom{3}{2}}{\binom{100}{2}} = \frac{1\cdot 3+3}{4950} = \frac{6}{4950} = \frac{1}{825}.$$

Furthermore,

$$\Pr(X = 10) = \frac{\binom{4}{1}\cdot\binom{6}{1}+\binom{6}{2}}{\binom{100}{2}} = \frac{4\cdot 6+15}{4950} = \frac{39}{4950} = \frac{13}{1650}$$

(one \$10 ticket and one higher amount ticket, or two \$10 tickets),

$$\Pr(X=3)=\frac{\binom{10}{1}\cdot\binom{15}{1}+\binom{15}{2}}{\binom{100}{2}}=\frac{10\cdot 15+105}{4950}=\frac{255}{4950}=\frac{17}{330},$$

(one \$3 ticket and one higher amount ticket, or two \$3 tickets),

and finally

$$\Pr(X=0)=\frac{\binom{25}{1}\cdot\binom{75}{1}+\binom{75}{2}}{\binom{100}{2}}=\frac{25\cdot 75+2775}{4950}=\frac{4650}{4950}=\frac{31}{33}$$

(for $X=0$, one \$0 ticket and one other ticket, or two \$0 tickets).

Thus

$$E(X)=25\cdot\frac{1}{825}+10\cdot\frac{13}{1650}+3\cdot\frac{17}{330}+0\cdot\frac{31}{33}=$$

$$=\frac{1}{33}+\frac{13}{165}+\frac{17}{110}=\frac{10}{330}+\frac{26}{330}+\frac{51}{330}=\frac{87}{330}\approx 0.2636.$$

Answer C.