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<http://smartURL.it/krzysioP> (paper) or <http://smartURL.it/krzysioPe> (electronic)

Instructor for Course P/1 online seminar: <http://smartURL.it/onlineactuary>

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Questions about these exercises? E-mail: krzysio@krzysio.net

Exercise for April 16, 2005

You are given that X and Y both have the same uniform distribution on $[0, 1]$, and are independent. $U = X + Y$ and $V = \frac{X}{X + Y}$. Find the joint probability density function of

(U, V) evaluated at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.

- A. 0 B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. 1

Solution.

We have $X = UV$, and $Y = U - UV$. Therefore,

$$\frac{\partial(x, y)}{\partial(u, v)} = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \det \begin{bmatrix} v & u \\ 1 - v & -u \end{bmatrix} = -uv - u + uv = u.$$

This implies that

$$f_{U, V}(u, v) = f_{X, Y}(x(u, v), y(u, v)) \cdot \left| \frac{\partial(x, y)}{\partial(u, v)} \right| = 1 \cdot 1 \cdot |u| = u.$$

This density at $\left(\frac{1}{2}, \frac{1}{2}\right)$ equals $\frac{1}{2}$.

Answer D.

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