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<http://smartURL.it/krzysioP> (paper) or <http://smartURL.it/krzysioPe> (electronic)

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Questions about these exercises? E-mail: [krzysio@krzysio.net](mailto:krzysio@krzysio.net)

Exercise for April 16, 2005

You are given that  $X$  and  $Y$  both have the same uniform distribution on  $[0, 1]$ , and are independent.  $U = X + Y$  and  $V = \frac{X}{X + Y}$ . Find the joint probability density function of

$(U, V)$  evaluated at the point  $\left(\frac{1}{2}, \frac{1}{2}\right)$ .

- A. 0      B.  $\frac{1}{4}$       C.  $\frac{1}{3}$       D.  $\frac{1}{2}$       E. 1

Solution.

We have  $X = UV$ , and  $Y = U - UV$ . Therefore,

$$\frac{\partial(x, y)}{\partial(u, v)} = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \det \begin{bmatrix} v & u \\ 1 - v & -u \end{bmatrix} = -uv - u + uv = u.$$

This implies that

$$f_{U, V}(u, v) = f_{X, Y}(x(u, v), y(u, v)) \cdot \left| \frac{\partial(x, y)}{\partial(u, v)} \right| = 1 \cdot 1 \cdot |u| = u.$$

This density at  $\left(\frac{1}{2}, \frac{1}{2}\right)$  equals  $\frac{1}{2}$ .

Answer D.

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