1. Use \( f(x) = 2x^2 - 3x + 1 \) to respond to (a) through (d).
   a. Calculate \( f'(x) \).
   b. Determine an equation for the line tangent to the graph of \( f \) when \( x = -1 \).
   c. Determine all values of \( x \) that lead to a horizontal tangent line.
   d. Determine all ordered pairs of \( f \) for which \( f'(x) = 1 \).

2. Suppose \( s(x) = 3x^4 - 2x^2 + \frac{s}{x} \) represents an object’s position as it moves back and forth on a number line, with \( s \) measured in centimeters and \( x \) in seconds, for \( x > 0 \).
   a. Calculate the object’s velocity and acceleration functions.
   b. Is the object moving left or right at time \( x = 1 \)? Justify.
   c. Determine the object’s velocity and acceleration at time \( x = 2 \). Based on those results, describe everything you can about the object’s movement at that instant.
   d. Write an equation for the tangent line to the graph of \( s \) at time \( x = 1 \).

3. Use \( g(x) = x\sqrt{x} \) to respond to (a) through (c).
   a. Determine the equation for the line tangent to the graph of \( g \) at \( x = 4 \).
   b. Determine the equation for the line normal to the graph of \( g \) at \( x = 1 \).
   c. At what points on the graph of \( g \), if any, will a tangent line to the curve be parallel to the line \( 3x - y = -5 \)?

4. Use \( h(t) = \cos(2t) \) to respond to (a) through (e).
   a. Calculate \( h'(t) \) and \( h''(t) \).
   b. Determine an equation for the line tangent to the graph of \( h \) when \( t = \pi/8 \).
   c. Determine the two values of \( t \) closest to \( t = 0 \) that lead to horizontal tangent lines.
   d. Determine the smallest positive value of \( t \) for which \( h'(t) = 1 \).
   e. If \( h(t) \) represents an object’s position on the number line at time \( t \) (\( h \) in feet, \( t \) in minutes), calculate the object’s velocity and acceleration at time \( t = \pi/12 \). Based on those results, describe everything you can about the object’s movement at that instant.
1. \( f(x) = 2x^2 - 3x + 1 \)
   a) \( f'(x) = 4x - 3 \)
   b) at \( x = -1 \): \( f(-1) = 2(-1)^2 - 3(-1) + 1 = 6 \)
      at \( x = -1 \): \( f'(-1) = 4(-1) - 3 = -7 \)
      So point is \((-1, 6)\) with slope \( m = -7 \)
      \[ y - 6 = (-7)(x - (-1)) \]
      \[ y = -7x - 1 \]
   c) horizontal tangent \( \Rightarrow f'(x) = 0 \)
      \( \Rightarrow \) seek \( x \) so that \( 4x - 3 = 0 \) \( \Rightarrow x = \frac{3}{4} \)
      \( f'\left(\frac{3}{4}\right) = 0 \Rightarrow \) horizontal tangent line
   d) \( f'(x) = 1 \) \( \Rightarrow 4x - 3 = 1 \) \( \Rightarrow x = 1 \)
      if \( x = 1 \), \( f(1) = 0 \)
      desired ordered pair: \((1, 0)\)
(2) \( s(x) = 3x^4 - 2x^2 + \frac{5}{x} \) (position on number line, \( x > 0 \))

\( a) \) velocity: derivative of position, so
\[ u(t) = s'(t) = 12x^3 - 4x - \frac{5}{x^2} \]
acceleration: derivative of velocity, second derivative of position
\[ a(t) = v'(t) = s''(t) = 36x^2 - 4 + \frac{10}{x^3} \]

\( b) \) Right Movement: \( u > 0 \); Left Movement: \( u < 0 \)

at \( x = 1 \):
\[ u(1) = 12 - 4 - 5 = 3, \text{ so } u(1) > 0 \Rightarrow \text{ object moving to the right.} \]

\( c) \) at \( x = 2 \):
\[ u(2) = \frac{347}{4} > 0 \Rightarrow \text{ object moving right} \]
\[ a(2) = \frac{565}{4} \]
acceleration is the derivative of velocity, so when \( a > 0 \) and \( u > 0 \), the object is moving right \( (u > 0) \) and the velocity is increasing \( (a > 0) \). Thus, with \( s(2) = \frac{85}{2} \), at \( x = 2 \) we know the object is at the point \( \frac{85}{2} \) on the number line, the object is moving right \( (u > 0) \), and it is moving at a faster rate \( (a > 0) \) in that direction.

\( d) \) at \( x = 1 \):
\[ s(1) = 6, \ s'(1) = u(1) = 3, \text{ so} \]
\[ y - 6 = 3(x - 1) \Rightarrow y = 3x + 3 \]
3. \( g(x) = x\sqrt{x} = x \cdot x^{\frac{1}{2}} = x^{\frac{3}{2}} \)

a) tangent line at \( x=4 \):

Need \( g'(x) = \frac{3}{2} \cdot x^{\frac{1}{2}} = \frac{3}{2} \cdot \sqrt{x} \)

So \( g'(4) = \frac{3}{2} \cdot \sqrt{4} = 3 \)

and \( g(4) = 4 \cdot \sqrt{4} = 8 \)

So \( y - 8 = 3(x - 4) \Rightarrow y = 3x - 4 \)

b) normal to graph at \( x=1 \).

Note: Whereas the tangent line at a point matches (is equal to) the slope of the curve, the normal line has slope perpendicular to the slope at the point. Recall that any two non-zero slopes for perpendicular lines are opposite reciprocals of each other.

So, at \( x=1 \), \( g'(1) = \frac{3}{2} \cdot \sqrt{1} = \frac{3}{2} \), so the normal line has slope of \( \frac{(-1)}{g'(1)} = \frac{-1}{\frac{3}{2}} = -\frac{2}{3} \). We need a point, \( g(1) = (1)\sqrt{1} = 1 \), so slope is \(-\frac{2}{3}\) and point is \((1,1) \Rightarrow y - 1 = (-\frac{2}{3})(x - 1) \)

or \( y = (-\frac{2}{3})(x) + \frac{5}{3} \)
(3-continued)

(c) Where on \( g \) is tangent line parallel to \( 3x - y = -5 \)?

The line \( 3x - y = -5 \), or \( y = 3x + 5 \), has slope of 3. So we seek points on \( g \) with slope 3: \( g'(x) = \frac{3}{2} \sqrt{x} \), so we seek \( x \) to make \( g'(x) = 3 \):

\[
\frac{3}{2} \sqrt{x} = 3 \implies \sqrt{x} = 2 \implies x = 4
\]

And \( g(4) = 4\sqrt{4} = 4 \cdot 2 = 8 \). Therefore, \( g(x) \) is parallel to \( 3x - y = -5 \) at the point \((4, 8)\).
(4) \( h(t) = \cos(2t) \)

\[ a) \quad h'(t) = -\sin(2t) \cdot 2 = -2\sin(2t) \]
\[ h''(t) = \frac{d}{dt}(-2\sin(2t)) = (-2\cos(2t)) \cdot 2 \]
\[ = -4\cos(2t) \]

\[ b) \text{ tangent line equation at } t = \frac{\pi}{8}: \]
- At \( t = \frac{\pi}{8} \), \( h(t) = h\left(\frac{\pi}{8}\right) = \cos\left(2 \cdot \frac{\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \)
- At \( t = \frac{\pi}{8} \), \( h'(t) = h'\left(\frac{\pi}{8}\right) = -2\sin(2 \cdot \frac{\pi}{8}) = -2\sin\left(\frac{\pi}{4}\right) = -\sqrt{2} \)

So the point is \( \left(\frac{\pi}{8}, \frac{\sqrt{2}}{2}\right) \) and slope is \( -\sqrt{2} \):

\[ y - \frac{\sqrt{2}}{2} = (-\sqrt{2})(x - \frac{\pi}{8}) \]
\[ \Rightarrow \quad y = -\sqrt{2}x + \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} \]

or

\[ y = -\sqrt{2}x + \frac{\sqrt{2}(\pi + 4)}{8} \]

c) horizontal tangent \( \Rightarrow \) slope is 0 \( \Rightarrow \) \( h'(t) = 0 \)

So we seek \( t \), \( t \) close to 0, such that
\[ h'(t) = 0 \quad \Rightarrow \quad h'(t) = -2\sin(2t), \quad \text{so calculate } t; \]
\[ 0 = -2\sin(2t) \Rightarrow \sin(2t) = 0 \quad \Rightarrow \quad \sin(t) = 0 \quad \text{if } \]
\( t = \pi k, \text{ an integer} \), so \( k = 1, k = -1 \) give no values \( t \).
(4 con't)

(c - con't)

So \( k = \pm 1 \) give us values close to \( t = 0 \),
and \( 2t = \pi \) \((k=1)\) \(\Rightarrow t = \frac{\pi}{2} \)
and \( 2t = -\pi \) \((k=-1)\) \(\Rightarrow t = -\frac{\pi}{2} \).

So the values closest to \( t = 0 \) that give us horizontal tangents occur at \( t = \pm \frac{\pi}{2} \).

(d) \( h'(t) = 1 \Rightarrow -2 \sin(2t) = 1 \Rightarrow \)
\( \sin(2t) = -\frac{1}{2} \); we know \( \sin(u) = -\frac{1}{2} \)
for \( u = \frac{7\pi}{6} \) \( (u > 0) \), so \( 2t = \frac{7\pi}{6} \) \(\Rightarrow \)
\( t = \frac{7\pi}{12} \) is the smallest positive \( t \) value

generating \( h'(t) = 1 \).

(e) \( v(t) = h'(t) = -2 \sin(2t) \) \((\text{see} \ (4a))\)
\( a(t) = h''(t) = -4 \cos(2t) \) \((\text{see} \ (4a))\)

So \( v(\frac{\pi}{2}) = -2 \sin(2 \cdot \frac{\pi}{2}) = -2 \sin(\pi) = -2(0) = -0 \) \( \text{ft/sec} \)
\( a(\frac{\pi}{2}) = -4 \cos(2 \cdot \frac{\pi}{2}) = -4 \cos(\pi) = -4(-1) = 4 \) \( \text{ft/sec}^2 \)

So at \( t = \frac{\pi}{2} \), \( v < 0 \), \( a > 0 \); the object is moving to the left \( (v < 0) \) and velocity is decreasing \( (a > 0) \).
(4e-cont')

Focus more specifically on this

If $v < 0$ (and, therefore, object moving left) and $a < 0$ (velocity decreasing), this means velocity is moving further & further left of 0 -> meaning the speed of the object is picking up; the object is moving "faster & faster" in a negative (left) direction.

(look at graphs of $h$, $\dot{h}$, $\ddot{h}$) at the same time.