MAT 145

Test #5 Test Part I: No Calculators!

Do not use notes, calculators, other people's work, or any other aids.

Name: ____________________________________________ Date: ______________________

1. Determine the exact value of the definite integral \( \int_{0}^{1} (5 + 6x + 12x^2) \, dx \).

\[
\int_{0}^{1} (5 + 6x + 12x^2) \, dx = 5x + 3x^2 + 4x^3 \bigg|_{0}^{1} = 7(5 + 3 + 4) - 0 = 12
\]

2. A particle moves along the horizontal axis with acceleration \( a(t) = 2t, \ 0 \leq t \leq 10 \). We also know the initial velocity, \( v(0) = 0 \).

(a) Determine \( v(t) \), a function for the velocity.

\( v(t) = t^2 + C \), but \( v(0) = 0 \) \( \Rightarrow \) \( C = 0 \)

(b) Calculate the total distance traveled by the particle for \( 0 \leq t \leq 10 \).

Any other change direction on \( 0 \leq t \leq 10 \) \( \Rightarrow \) so... for total distance we need \( \int_{0}^{10} v(t) \, dt = \int_{0}^{10} t^2 \, dt = \frac{1}{3} t^3 \bigg|_{0}^{10} = \frac{1000}{3} \) units.

3. If \( g(x) = \int_{2}^{x} t^4 \cos(3t) \, dt \), determine \( g'(x) \).

\( g'(x) = v(t) \cos(3t) \)

4. Evaluate \( \int_{3}^{5} \frac{x^3 + 6x}{x} \, dx \).

\[
\int_{3}^{5} \frac{x^3 + 6x}{x} \, dx = \int_{3}^{5} (x^2 + 6) \, dx = \left[ \frac{1}{3} x^3 + 6x \right]_{3}^{5} = \frac{125}{3} + 30 - 27 = \frac{125}{3} + 9 = \frac{134}{3}
\]

5. We know that \( \int_{3}^{5} f(x) \, dx = -6 \), \( \int_{3}^{8} f(x) \, dx = 14 \), \( \int_{5}^{10} f(x) \, dx = 23 \). Determine \( \int_{3}^{10} f(x) \, dx \).

\[
\int_{3}^{10} f(x) \, dx = \int_{3}^{5} f(x) \, dx + \int_{5}^{8} f(x) \, dx + \int_{8}^{10} f(x) \, dx = -6 + 14 + 23 = 31
\]

Evaluation Criteria: 50 points (5 pts each)

#2: (a) 2 pts (b) 3 pts; #10: (a) 2 pts (function, rectangles, labels) (b) 2 pts (complete expression, correct symbols/organization) (c) 1 pt (correct common fraction)

Bonus!!! 5 pts: correct number of subdivisions with complete and clear justification
6. Use a Riemann Sum to approximate the area under the curve \( f(x) = \frac{1}{3 \cdot 2^x}, \ 0 \leq x \leq 2 \), using \( n = 4 \) subdivisions and the **midpoint method**. Round to the nearest thousandth.

\[
\Delta x = \frac{2 - 0}{4} = \frac{1}{2} \\
\text{Riemann Sum} = R_1 + R_2 + R_3 + R_4 \\
= \Delta x \cdot f(x_1) + \Delta x \cdot f(x_2) + \Delta x \cdot f(x_3) + \Delta x \cdot f(x_4) \\
= \left(\frac{1}{2}\right) \cdot f\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) f\left(\frac{3}{4}\right) + \left(\frac{1}{2}\right) f\left(\frac{5}{4}\right) + \left(\frac{1}{2}\right) f(2) \\
\approx 0.3589
\]

7. Let \( g(t) \) represent a child's rate of growth in pounds per year. Write a definite integral to represent the change in the child's weight, in pounds, from year 4 to year 9.

\[
\int_{4}^{9} g(t) \, dt
\]

8. An animal population is increasing at a rate of \( P'(t) = 4 + 29t \) animals per year, \( t \) measured in years. By how many animals does the population increase from year 3 to year 7?

\[
\int_{3}^{7} P'(t) \, dt = \int_{3}^{7} (4 + 29t) \, dt = 4t + \frac{29t^2}{2} \bigg|_{3}^{7} \\
= 147/2 - 38/2 = 596
\]

9. A particle moves along the horizontal axis with velocity \( v(t) = 3t - 3, \ 0 \leq t \leq 6 \), measured in ft/s. Determine the net change in position of the particle on \( 0 \leq t \leq 6 \).

\[
\text{Net Change} = \int_{0}^{6} v(t) \, dt = \int_{0}^{6} (3t - 3) \, dt \\
= \frac{3}{2} t^2 - 3t \bigg|_{0}^{6} = 36 \text{ ft}
\]
We wish to create a Riemann Sum to approximate the area under the curve \( y = x^2 + 2 \), \( 1 \leq x \leq 2 \), using \( n = 4 \) subdivisions and the left-endpoint method.

(a) Create a picture to show the function and the approximating rectangles. Clearly label the elements of your picture, including the function the horizontal axis scale, any x-axis tick marks and your rectangles.

\[
\Delta x = \frac{2-1}{4} = \frac{1}{4}
\]

\[
\text{left endpoints: } x_1 = 1, \quad x_2 = \frac{5}{4}, \quad x_3 = \frac{9}{4}, \quad x_4 = \frac{13}{4}
\]

(b) Write an expression, showing all terms, to represent the Riemann Sum.

\[
\text{Riemann Sum } = R_1 + R_2 + R_3 + R_4 = \Delta x \cdot f(x_1) + \Delta x \cdot f(x_2) + \Delta x \cdot f(x_3) + \Delta x \cdot f(x_4)
\]

\[
= (\frac{1}{4}) \cdot (3) + (\frac{1}{4})(5^2) + (\frac{1}{4})(17) + (\frac{1}{4})(31)
\]

\[
= 3 \frac{31}{32}
\]

(c) Calculate the exact value, expressed as a common fraction, for this Riemann Sum.

\[
\frac{13}{32} = 3.90625
\]

**Bonus!**

Let \( R \) be the first-quadrant region bounded by the x-axis, the y-axis, and the graph of \( y = 4 - x^2 \).

Determine the smallest number of subdivisions we must use in a left-endpoint Riemann Sum in order to approximate the actual area of \( R \) with error no more than 0.01. Describe your process, justification, and results.