4. Uncle Ed's Uniforms produces two styles of baseball uniforms. The modern-style uniforms each require 2 hours of labor to produce, while the retro-style uniforms each require 4 hours to produce. Uncle Ed's has 1122 hours of labor available each week to produce these two uniform types, and they plan to use all of those available hours. It is estimated that if they produce a total of $x$ modern-style uniforms each week and $y$ retro-style uniforms each week, then Uncle Ed's total weekly profit (in dollars) will be given by the function

$$P(x, y) = -2x^2 - 12y^2 + 12xy - 200x + 700y - 50,000.$$  

a) Write a constraint equation for this problem.

$$2x + 4y = 1122 \implies 2x + 4y - 1122$$

b) Determine the number of modern-style uniforms and the number of retro-style uniforms that should be produced in order to maximize Uncle Ed's weekly profit, subject to the constraint in part (a).

Use the Lagrange Multiplier Method

$$P_x(x, y) = -4x + 12y - 200 \quad P_y(x, y) = -24y + 12x + 700$$
$$C_x(x, y) = 2 \quad C_y(x, y) = 4$$

So

$$-4x + 12y - 200 = 2 \lambda \quad -24y + 12x + 700 = 4 \lambda$$

Solve each for $\lambda$ and eliminate $\lambda$ to get

$$-5x + 12y = 275; \text{ create system of equations using } P_x \text{ & constraint } C(x, y):$$

$$x + 2y = 561 \implies y = 140 \quad \text{ & } \quad x = 281$$

| $x$: Number of modern-style uniforms | 281 |
| $y$: Number of retro-style uniforms | 140 |

Maximum weekly profit

$P(281, 140) = \$40,322$
21. Woodworkers Incorporated has found that their marginal cost of producing $x$ feet of custom molding is given by $MC(x) = C'(x) = -0.00003x^2 - 0.04x + 45$ cents per foot ($0 \leq x \leq 600$).

a) Evaluate $C'(500)$. Label your answer with appropriate units. Interpret your answer in economic terms.

$C'(500) = 17.5$ cents/foot

At the instant that exactly 500 feet of custom fencing have been produced, the cost to produce an additional foot is 17.5 cents.

b) Sketch the graph of $C'(x) = -0.00003x^2 - 0.04x + 45$ for $0 \leq x \leq 600$ and copy it below.

Label each axis with appropriate units and label the value of the $y$-intercept.

c) Shade in the region that represent the definite integral $\int_{200}^{400} C'(x) \, dx$.

d) Use the fundamental theorem of calculus to evaluate $\int_{200}^{400} (-0.00003x^2 - 0.04x + 45) \, dx$ by hand. Do not use your calculator except to assist with arithmetic. You must show your work.

$$\int_{200}^{400} C'(x) \, dx = \frac{-0.00001x^3 - 0.02x^2 + 45x}{600} \bigg|_{200}^{400}$$

$$= 14160 - 8120 = 6040 \text{ cents}$$

$(or \ 60.40 \ \$)$