19. A zoo supply business expects to sell 1200 snakes during the next year, and they must pay $60 for each snake they buy. Due to the extreme care required in shipping the snakes, there is an $750 cost for each order they place. It costs the business $80 to care for one of the snakes for a year. Assume that the storage costs are calculated based on the average number of snakes.

a) Give an equation that represents the total inventory cost. State what each variable represents.

Be sure to show necessary steps that are used to derive your equation. Then indicate the interval (domain) over which this function would be minimized.

\[ X : \text{# of snakes per order} \quad \frac{X}{2} : \text{Average # snakes in storage} \]

\[ \text{Storage costs:} \quad (\$80/\text{snake}) \left( \frac{X}{2} \right) = \frac{40}{X} \]

\[ \text{# of orders:} \quad 1200 \quad \text{cost for all orders:} \quad (\$750)(1200) \]

Total inventory cost:

\[ C(X) = \text{Storage costs + Ordering costs + Product cost} \]

\[ = 40X + \left( \frac{750X}{2} \right) + (1200)(60) \]

For \( 1 \leq X \leq 1200 \)

b) How many snakes should the business order in each shipment to minimize the total inventory cost?

Use calculus to solve this problem. You must verify your answer yields a minimum inventory cost.

If \( C(X) = 40X + \left( \frac{750X}{2} \right) \), then

\[ C'(X) = 40 - \left( \frac{750X}{2} \right) \]

Set \( C'(X) = 0 \), solve for \( X \), \& verify:

\[ C'(X) = 0 \Rightarrow 40 - \left( \frac{750X}{2} \right) = 0 \]

\[ 40 - \left( \frac{750X}{2} \right) = 0 \Rightarrow X = \sqrt{\frac{750 \cdot 1200}{40}} \]

\[ X = \sqrt{22,500} = 150 \text{ snakes/order} \]

Verification

\( X = 150 \) is a potential location for a min of \( C(X) \). Show appropriate sign change in \( C'(X) \):

\[ C'(X) : \quad - \quad + \quad + \quad + \quad + \quad + \]

For \( X < 150 \), \( C'(X) < 0 \), \& for \( X > 150 \), \( C'(X) > 0 \), so, indeed, a min of \( C(X) \) occurs when there are \( X = 150 \) snakes per order.

Number of snakes per order \( X = 150 \) snakes

Number of orders per year \( \frac{1200}{150} = 8 \) orders

d) What is the minimum inventory cost?

\[ C(150) = \frac{40(150)}{150} + \left( \frac{750 \cdot 1200}{150} \right) + (1200)(60) \]

\[ = \$4,000 \]
Suppose that a maker of custom bicycles is willing to supply $x$ bikes per month at a price of $S(x) = 3x^2 + 10x$ dollars each, and has found that consumers are willing to purchase $x$ bikes per month at a price of $D(x) = -65x + 6300$ dollars each.

a) Sketch the graphs of the supply and demand function on the same axes and copy them below. Label the graphs, the $y$-intercepts, and label the axes with appropriate units. Make sure your graphs show the equilibrium point.

b) Find the equilibrium price and quantity. Be sure to label each with appropriate units.

Solve $S(x) = D(x)$ \Rightarrow $3x^2 + 10x = -65x + 6300$

\Rightarrow $x = -60$ or $x = 35$ (only use $x > 0$)

if $x = 35$, $S(35) = D(35) = 4025$

Equilibrium quantity \textbf{35 bikes} \hspace{1cm} \text{Equilibrium price} \textbf{\$4025/bike}$

c) Shade in the region on the graph above that represents the producers’ surplus.

d) Write an expression involving a definite integral that represents the producers’ surplus. Simplify your answer.

\text{See "P.S." shaded above.}

\int_{0}^{35} ((4025) - S(x)) \, dx = \text{P.S.}

e) Use your calculator to find the producers’ surplus by evaluating the expression in part (d).

\$9,187.5