Evaluate each indefinite integral. (2 pts each; no partial credit)

1) \( \int 3e^x \, dx \) 

2) \( \int 7x^8 - 8x^7 \, dx \) 

3) \( \int 3\sin\left(\frac{2x}{3}\right) \, dx \) 

4) \( \int \sec^2(x) \, dx \) 

5) \( \int \frac{3}{2x^4} \, dx \) 

6) \( \int \frac{3}{2x} \, dx \) 

7) \( \int \frac{5}{x^2 + 1} \, dx \) 

8) \( \int \frac{2}{\sqrt{1 - x^2}} \, dx \) 

9) \( \int x^{e^2+2} \, dx \) 

10) \( \int 11^x \, dx \)
Questions (11) through (16) are each worth 5 points. See the grading rubric for further details.

11. Evaluate the following definite integral: $\int_{\pi/4}^{\pi/2} (\cos^3 x) \, dx$.  

12. Evaluate the following indefinite integral: $\int (3x)(e^{2x}) \, dx$. Simplify and combine like terms in your final response.
13-(i) Consider the following integral: \( \int_{3}^{7} \frac{5}{x} \, dx \). Which of the following responses is the most accurate? Circle one. (2 pts)

(A) \( \int_{3}^{7} \frac{5}{x} \, dx = 5 \ln |x| \)  
(B) \( \int_{3}^{7} \frac{5}{x} \, dx = 5 \ln |x| + C \)  
(C) \( \int_{3}^{7} \frac{5}{x} \, dx = 5 \ln |4| \)  
(D) \( \int_{3}^{7} \frac{5}{x} \, dx = \ln |7| - \ln |3| \)  
(E) \( \int_{3}^{7} \frac{5}{x} \, dx \) does not exist  
(F) None of (A) through (E) is correct.

13-(ii) Buddy painted one side of a fence whose face is defined by the function \( f(x) = \frac{12}{(x + 2)^2} \) for \( 0 \leq x < \infty \), with \( x \) measured in feet. Calculate the exact measure of the surface area that Buddy painted. Show all steps leading to your solution. (3 pts)

14. Set up, but do not calculate, an integral expression to determine the arc length of the function \( f(x) = 2x^2 - 3 \), on the interval \( 2 \leq x \leq 5 \).
15. A slope field for a differential equation is shown here, including the sketch of a particular solution to that differential equation.

(i) Which of the following differential equations could not possibly be the source for this slope field? Circle all that apply. (2 pts)

(A) \( \frac{dy}{dx} = 2x^4 \) 
(B) \( \frac{dy}{dx} = (-3y^6)(e^{2x}) \) 
(C) \( \frac{dy}{dx} = -(\cos(xy))^2 \) 
(D) \( \frac{dy}{dx} = 2y^3 \)

(ii) Choose exactly one of the differential equations you circled for (i) and explain how you determined that the differential equation could not be the source for this slope field. Be specific and precise. (3 pts)

16. Which of the following series diverge?

I. \( \sum_{n=0}^{\infty} \frac{4^{10}}{3^{n+1}} \) 
II. \( \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{2n} \) 
III. \( \sum_{n=1}^{\infty} \frac{3(n-1)^2}{5e^2} \)

(a) Choose and circle one response from among (A) through (F). (2 pts)

(A) I only 
(B) II only 
(C) III only 
(D) I and II only 
(E) II and III only 
(F) None of these diverge.

Your Choice: A B C D E F (Circle one.)

(b) Identify one of these series (I, II, or III) and explain how you determined whether that series diverged. (3 pts)
**BONUS #1**

Let $R$ be the region enclosed by the graphs of $f(x) = ax(2 - x)$ and $g(x) = ax$ for some positive real number $a$. Show complete and appropriate evidence and justification.

(i) Calculate the exact area of region $R$. (3 pts)
(ii) Calculate the exact volume of the solid of revolution created when $R$ is rotated about the $x$-axis. (3 pts)
(iii) Assume that a solid exists with a cross-section area of $R$ and uniform thickness $\pi$. Determine the exact value of $a$ for which this solid has the same volume as the solid in (ii). (4 pts)

**BONUS #2**

A power series representation for $h(x)$ is given by $h(x) = 2x - 4x^2 + 6x^3 - 8x^4 + \cdots$.

Provide complete and appropriate evidence with your responses.

(i) Show a summation notation representation for $h(x)$. (2 pts)
(ii) Determine the interval of convergence for $h(x)$. (2 pts)

(iii) If $K(x) = \int h(x) \, dx$ and $K(0) = 0$, generate a power series, expressed using sigma notation, for $2h'(x) + K(x)$. Calculate, simplify, and show the first three non-zero terms (starting with a constant, if one exists) of that power series. (4 pts)