Part I: Do Not Use Any Calculator or Computer Tools!

Evaluate each indefinite integral. Do not use your graphing calculator. (2 pts each)

1) \( \int \pi e \, dx \)

2) \( \int 6x^3 - 3x^2 \, dx \)

3) \( \int 4\sec^2 x \, dx \)

4) \( \int e^x \, dx \)

5) \( \int \frac{1}{x^2 + 1} \, dx \)

6) \( \int 17^x \, dx \)

7) \( \int 2\cos(3x) \, dx \)

8) \( \int \frac{4}{x^7} \, dx \)

9) \( \int \frac{2}{\sqrt{1-x^2}} \, dx \)

10) \( \int \frac{1}{2x} \, dx \)
Questions (11) through (16) are each worth 5 points. See the grading rubric for further details.

11. Evaluate the following indefinite integral: \( \int (\sin^2 x)(\cos x) \, dx \). 

12. Evaluate the following definite integral: \( \int_{0}^{4} (\sin x)(e^{2x}) \, dx \). Simplify and combine like terms in your final response.
13. Determine whether the following statement is TRUE or FALSE: \[ \int_{-2}^{1} \frac{1}{x^4} = -\frac{3}{8}. \]  

   TRUE    FALSE  (Circle one.)

Explain or show evidence to support your response.

14. Set up, but do not calculate, an integral expression to determine the volume of the solid of revolution determined when the first-quadrant region enclosed by \( f(x) = x^2 \), \( g(x) = 4 \), and the y-axis, rotated about the line \( y = -1 \). Include a sketch to represent the situation.
15. **Exactly one** of the functions shown below, (A) through (E), is a solution to the differential equation \( \frac{dy}{dx} = 3x^2 e^{-2y} \) where \( y(1) = 4 \). Identify the correct solution among (A) through (E) and justify your choice.

\[
\begin{align*}
(A) & \quad y = \frac{1}{2} \ln(2x^3) \\
(B) & \quad y = \frac{1}{2} \ln(2x^3) - \ln\sqrt{2} + 4 \\
(C) & \quad y = \frac{1}{2} \ln\left(\frac{1}{2} x^3 + e^8 - \frac{1}{2}\right) \\
(D) & \quad y = \frac{1}{2} \ln\left(2x^3 + e^8 - 2\right) \\
(E) & \quad y = \frac{1}{2} \ln\left(x^3 + e^8 - 1\right)
\end{align*}
\]

Correct Solution: A B C D E (Circle one.)

Justify:

16. Which of the following series, expressed as functions, include \(-1 \leq x \leq 1\) in their domains? Explain and justify your determination. Choose one response, (A) through (F).

\[
\begin{align*}
&I. \quad f(x) = \sum_{n=1}^{\infty} \frac{1}{(3x)^n} \\
&II. \quad g(x) = \sum_{n=1}^{\infty} \frac{2^n}{n^3} x^n \\
&III. \quad h(x) = \sum_{n=1}^{\infty} \frac{x^n}{(2n)!}
\end{align*}
\]

(A) I only \quad (B) II only \quad (C) III only
(D) I and II only \quad (E) II and III only \quad (F) I, II, and III

Correct Choice: A B C D E F (Circle one.)

Explain and justify:
BONUS: Part 1

Here are two bonus problems. You may complete one of these for extra credit. Show your response in the space below.

(A) The curve defined by $y^2 - 6xy + 6x^2 = 1$ has two different tangent lines to the curve where $x = 1$. Determine the point of intersection of these two tangent lines. Express your solution as an ordered pair.

(B) The rate of decay of radioactive uranium is proportional to the amount $A$ of uranium present at any time $t$, $t$ in years. If there are initially 54 grams of uranium present, and there are 42 grams present after 120 years, represent this situation as a function $A(t)$. 
