Questions (17) through (26) are each worth 5 points. See the grading rubric for further details.

17. The graph of a polynomial function $f$ is shown here. Use it to determine responses to the following questions. Write equations, such as $x = f$ or $y = g$, or inequalities in $x$ or $y$, as your responses.

(a) State the global maximum of $f$.

__________________________

(b) State the positive zeros (or positive roots) of $f$.

__________________________

(c) State all intervals over which $f$ is decreasing.

__________________________

(d) State all intervals over which $f$ is concave up.

__________________________

(e) State all intervals over which $f'$, the derivative of $f$, is positive.

__________________________

18. Use the Trapezoid Rule with $n = 3$ subdivisions to approximate $\int_{0}^{3} e^{-x^2} dx$. State the exact value of your approximation as a rational expression.

__________________________
19. Determine the area of the region in the 3\textsuperscript{rd} quadrant bounded by the curves

\[ y = x, \, y = \frac{3}{x-2}, \text{ and } x = 0. \]

Include a sketch to illustrate the situation, show your steps leading to solution, and express your solution as an exact value.

20. Assume that the daily consumption of water (in millions of gallons) for a metropolitan region has the following probability density function:

\[ p(x) = \begin{cases} 
\frac{1}{9}xe^{-\frac{x}{9}} & \text{if } x \geq 0 \\
0 & \text{if } x < 0 
\end{cases} \]

(a) What is the probability that the region’s daily water consumption will range between 2 million and 4 million gallons? Round to the nearest thousandth of a unit, if necessary.

(b) If the region’s water supply has a daily capacity of 10 million gallons, what is the probability that the available water supply will be inadequate on any given day? Round to the nearest thousandth of a unit, if necessary.

(c) State the two characteristics of \( p(x) \) that assure it is a probability density function.
21. For the series \( \sum_{n=1}^{\infty} a_n \), if \( \lim_{n \to \infty} a_n = 0 \), we can conclude that the series _?_ converge.

(i) Choose the best word or phrase to correctly complete this sentence:

(A) will (B) will not (C) may or may not  

(Circle one.)

(ii) For the choice you made in (i), provide an example to illustrate your selection. If you selected (A) or (B), you need to provide _one_ illustration. If you chose (C), you need _two_.

22. State and solve an inequality, involving \( d \), to describe the conditions under which the series \( \sum_{m=1}^{\infty} \frac{2}{m^{3-d}} \) will converge.

\[ \text{ } \]

23. Which of the three infinite series shown here will converge? (Circle the most appropriate response) from (A) through (H) and then _briefly explain_ your choice. _Formal proof is not required._

\( \sum_{n=1}^{\infty} \frac{2^n}{5^{n+2}} \) \( \sum_{n=1}^{\infty} \frac{n}{2n^2 + 1} \) \( \sum_{n=1}^{\infty} (-1)^n (2n)^{-2} \)

A) None  B) only I  C) only II  D) only III  
E) only I and II  F) only I and III  G) only II and III  H) I, II, and III
24. Determine whether the series \( \sum_{n=0}^{\infty} \frac{3^{n+2}}{5^n} \) is convergent or divergent. Provide complete and appropriate justification for your response by using one or more tests for convergence. Tell your story!

25. Determine the radius of convergence and the interval of convergence for the power series given by \( f(x) = \sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{4^n} \). Use appropriate notation and provide complete justification for your responses.

radius of convergence: _______________________

interval of convergence: ______________________
Consider the function $f$ defined by the power series $f(x) = \sum_{n=1}^{\infty} \frac{nx^{n-1}}{3^n}$ for all $x$ in the interval of convergence.

(a) Determine the radius of convergence.

(b) Let $g$ be the function such that $g'(x) = f(x)$. That is, $f$ is the derivative of $g$, with $g(1) = 3$. Determine the first three non-zero terms and the general term for the series representation of $g(x)$.

(c) If possible, evaluate $g(3)$. If this is not possible, explain why not.
**BONUS: Part 2**

Here are two bonus problems. You may complete one of these for extra credit. Show your response in the space below. Continue to the back side as necessary. Clearly indicate your responses.

(C)

(i) Write the Taylor polynomial centered at $x = 0$ for $g(x) = xe^x$. Show the first three terms and write an expression using summation notation. Justify your response.

(ii) Write the Taylor polynomial centered at $x = 0$ for $\int xe^x dx$. Show the first three terms and write an expression using summation notation. Justify your response.

(iii) Determine the exact value of $\int_0^1 xe^x dx$ using integration by parts. Show all steps.

(iv) Show how to use the results of (ii) and (iii) to determine the exact value of $\sum_{n=1}^{\infty} \frac{1}{(n+2)n!}$. Justify your response.

(D)

(i) Expand $f(x) = \frac{x}{(1-x)^2}$ as a power series. Show the first three terms and write an expression using summation notation. Justify your response.

(ii) Show how use the results of (i) to determine the exact numerical value for $\sum_{n=1}^{\infty} \frac{n}{2^n}$. 

Calculus II
MAT 146
Semester Exam

Total Points: 100
Impact of Exam on Semester Grade: Approximately 25%

Evaluation Criteria

Unless otherwise directed, state any numerical solutions as exact values in rational expressions reduced to lowest terms. If approximations are required, express as a decimal value rounded accurately to the nearest thousandth of a unit.

Part I: No Calculators
Questions 1 through 10
2 points each with no partial credit. No need to show work on these.

Questions 11 through 16
5 points each. Partial credit is possible. Show all steps leading to your solutions. Be clear, complete, and accurate.

11) 3 pts: complete steps to solution; 2 pts: correct solution
12) 3 pts: complete steps to solution; 1 pt: correct integral evaluation; 1 pt: correct numerical solution
13) 1 pt: correct TRUE/FALSE response; 4 pts: complete and accurate explanation/evidence
14) 4 pts: complete and accurate integral set-up; 1 pt: appropriate sketch
15) 2 pts: correct choice; 3 pts: complete and accurate justification
16) 2 pts: correct choice; 3 pts: complete and accurate justification

Part 1 Bonus: 5 pts: Complete, accurate, and justified response. Select (A) or (B).
(A) correct statement of intersection point, as an ordered pair, with complete and accurate justification
(B) correct statement of required functional relationship \( A(t) \), with required evidence justifying your function

Part II: Calculators Allowed
Questions 17 through 26 are worth 5 points each. Partial credit is possible. Show all steps leading to your solutions. Be clear, complete, and accurate.

17) 1 pt each, using correct format (equation or inequality)
18) 3 pts: correct set-up and application of the Trapezoid Rule; 2 pts: correct exact-value numerical approximation
19) 2 pts: complete steps to solution; 2 pts: correct numerical result; 1 pt: appropriate sketch
20) (a) and (b) 2 pts each: correct numerical solutions; (c) 1 pt: correct two criteria
21) (i) 2 pts correct response (ii) 3 pts appropriate illustration, consistent with your response to (i)
22) 5 pts for correct inequality involving \( d \)
23) 2 pts: correct response choice; 3 pts: explanation
24) 1 pt: correctly state convergence or divergence; 4 pts: complete and appropriate justification in narrative format
25) 3 pts: radius of convergence with evidence; 2 pts: interval of convergence with evidence
26) (a) 1 pt: radius of convergence with evidence (b) 2 pts: correct first three terms and general term; (c) 2 pts: correct value of \( g(3) \) OR explanation for non-existence of \( g(3) \)

Part 2 Bonus: 5 pts: Complete, accurate, and justified response. Select (C) or (D).
(C) (i) – (iii) 1 pt each, correct response with justification; (iv) 2 pts: correct response with justification
(D) (i) 2 pts, correct response with justification; (ii) 3 pts: correct response with justification