17. The graph of a 7th-degree polynomial function \( y = w(x) \) is shown here. Use it to determine responses to questions (a) through (e). **Respond with equations, such as \( x = m \) or \( y = ii \), ordered pairs \((x,y)\), inequalities in \( x \) or \( y \), or integer values.** (1 pt each)

(a) Use an **ordered pair** to state the location of one third-quadrant local minimum of \( w \).

__________________________

(b) Use an **ordered pair** to state the \( y \)-intercept of the graph of \( w \).

__________________________

(c) State one **negative x-axis interval** over which \( w \) is **decreasing**.

__________________________

(d) State one **positive x-axis interval** over which \( w \) is **concave down**.

__________________________

(e) Among the function’s **points of inflection**, use an **ordered pair** to state the point of inflection with the **least y value**.

__________________________

18. The curve \( H \), graphed here, is identified by the equation \( 3xy + y^2 = x^2 + 13 \). Momjab claims that \( H \) is a solution to the differential equation \( \frac{dy}{dx} = \frac{2x - 3y}{3x + 2y} \).

(a) Determine every ordered pair \((x,y)\) at which \( H \) will have a horizontal tangent. Provide evidence for your findings. (3 pts)

(b) Imagine (or sketch one here!) a slope field associated with Momjab’s differential equation. Use that image to write the equation of the line in the \( xy \)-plane that holds all the ordered pairs where the slope field would show undefined slope. (2 pts)
19. Determine the exact area of the region $R$ in the 1st quadrant bounded by the curves 

$$y = \frac{1}{2}x^2, \quad y = -x + 4, \quad \text{and} \quad x = 0.$$ 

Create a sketch to illustrate the situation, including clear and accurate labels for all borders and for the region $R$. Show all steps leading to your solution. Express your solution as an exact value.

\[ \text{________________________} \]

20. The probability density function representing the distribution of the ages of babies being brought to a postnatal clinic, where $x$ is in years of age, is given by

$$f(x) = \begin{cases} \frac{3}{4}x(2-x) & \text{for } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the probability that the next baby brought to the clinic will be less than 3 months old? Show and calculate the value of a definite integral. Round to the nearest thousandth of a unit, if necessary. (2 pts)

\[ \text{________________________} \]

(b) Suppose 50 babies are brought to the clinic some week. Based on the density function $f(x)$, estimate the number of these babies that are older than 18 months of age. Show evidence. (2 pts)

\[ \text{________________________} \]

(c) An infant wellness clinic in a different city has $g(x)$ as its probability density function

$$g(x) = \begin{cases} Ax^2(4-x) & \text{for } 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

where $g(x)$ represents the distribution of the ages of infants brought in, with $x$ in years of age. Determine the exact value of $A$. (1 pt)

\[ \text{________________________} \]
21. For the series $\sum_{n=1}^{\infty} a_n$, if $\lim_{n \to \infty} a_n = 0$, we can conclude that the series _?_ diverge.

(i) Choose the best word or phrase to correctly complete the sentence: (2 pts)

(A) will \hspace{1cm} (B) will not \hspace{1cm} (C) may or may not \hspace{1cm} (Circle one.)

(ii) For the choice you just made, provide example series to illustrate your decision. If you selected (A) or (B), you need one series. If you chose (C), you need two series (3 pts)

________________________________________________________________________

22. Determine all values of $w$ for which the series $\sum_{n=1}^{\infty} \frac{1}{m^{w+3}}$ will converge. Begin by stating an inequality based on the series presented here.

________________________________________________________________________

23. Which of the three infinite series shown here will diverge? Circle the most appropriate response from (A) through (H) (2 pts). After making your selection, briefly explain the basis of your decision for ONE of the series.

(I) $\sum_{n=1}^{\infty} \frac{3n}{n^2+1}$ \hspace{1cm} (II) $\sum_{n=1}^{\infty} (-1)^{n+1} (n)^{-2}$ \hspace{1cm} (III) $\sum_{n=1}^{\infty} \frac{4^{n-1}}{5^{n+1}}$

A) None \hspace{1cm} B) only I \hspace{1cm} C) only II \hspace{1cm} D) only III
E) only I and II \hspace{1cm} F) only I and III \hspace{1cm} G) only II and III \hspace{1cm} H) I, II, and III

From among the three series (I, II, and III), select one and explain your convergence/divergence decision. (3 pts)
24. For the series \( \sum_{n=0}^{\infty} \frac{2^n}{5^{n+1}} \):

(a) Prove that the series is convergent. Provide complete and appropriate justification for your response by using one or more tests for convergence. (3 pts)

(b) Determine the exact value (sum) of the series. Explain and justify your result. (2 pts)

25. Determine the radius of convergence and the interval of convergence for the power series given by \( h(x) = \sum_{n=0}^{\infty} \frac{(2x + 1)^n}{2^n} \). Use appropriate notation and provide complete justification for your responses.

- interval of convergence: _________________ (3 pts)
- radius of convergence: _________________ (2 pts)
26-I. We know that the power series \( \sum_{n=0}^{\infty} x^n \) represents the function \( g(x) = \frac{1}{1-x} \) on \(-1 < x < 1\).

Show the \textit{sigma notation representation (general term)} for the power series that represents each of the following functions. (1 pt each)

(i) \( h(x) = \frac{1}{1+x} \)

(ii) \( k(x) = \frac{2}{1+x^2} \)

(iii) Let \( p \) be the function such that \( p(x) = g'(x) \). That is, \( p \) is the derivative of \( g \), where \( g(x) = \frac{1}{1-x} \). Determine \textit{the sigma notation} representation for the power series that represents \( p(x) \).

(ii)_____________________________(1 pt)

26-II.

(iv) Write the 8\textsuperscript{th}-degree Taylor Series expansion for \( f(x) = e^{x^2} \) centered at \( x = 0 \).

(iv)_____________________________(1 pt)

(v) Use your answer to (iv) to accurately approximate the definite integral \( \int_{0}^{1} e^{x^2} \, dx \). Show your exact-value calculation as a common fraction.

(v)_____________________________(1 pt)
The series \( \sum_{n=2}^{\infty} \frac{5}{n^2 - n} \) appears to converge. Show how to use a **telescoping sum** to both **justify** that the series converges and to **show the exact value** of the sum. Provide complete and appropriate evidence. (5 pts)
## Evaluation Criteria

Unless otherwise directed, state any numerical solutions as exact values in rational expressions reduced to lowest terms. If approximations are required, express as a decimal value rounded accurately to the nearest thousandth of a unit.

### Part I: No Calculators
50 points

Questions 1 through 10
2 points each with no partial credit. No need to show work on these.

Questions 11 through 16
5 points each. Partial credit is possible. Show all steps leading to your solutions. Be clear, complete, and accurate.

11) 3 pts: complete steps to solution; 2 pts: correct solution  
12) 3 pts: complete steps to solution; 1 pt: correct integral evaluation; 1 pt: correct numerical solution  
13) (i) 2 pts: correct response choice; (ii) 3 pts: correct exact-value SA calculation with appropriate evidence  
14) 5 pts: complete and accurate integral set-up  
15) (i) 2 pts: correct response choice; (ii) 3 pts: complete and appropriate explanation  
16) 2 pts: correct choice; 3 pts: complete and accurate justification for one power series

### Part II: Calculators Allowed
50 points

Questions 17 through 26 are worth 5 points each. Partial credit is possible. Show all steps leading to your solutions. Be clear, complete, and accurate.

17) 1 pt each, using correct format (ordered pair or inequality)  
18) (a) 3 pts: correct ordered pairs and evidence; (b) 2 pts: correct linear equation and evidence  
19) 2 pts: complete steps to solution; 1 pt: correct numerical result; 2 pts: appropriate sketch with all required labels  
20) (a) and (b) 2 pts each: correct numerical solutions with evidence; (c) 1 pt: correct value for A  
21) (i) 2 pts: correct response (ii) 3 pts: appropriate series illustration, consistent with your response to (i)  
22) 5 pts: correct inequality and solution involving w  
23) 2 pts: correct response choice; 3 pts: explanation  
24) 3 pts: complete and accurate convergence proof; 2 pts: accurate determination of sum, with evidence  
25) 3 pts: interval of convergence with evidence; 2 pts: radius of convergence with evidence  
26) (i-iii) 1 pt each: correct sigma notation (iv) 1 pt: correct and complete Taylor Series expansion (v) 1 pt: correct calculation expressed as a common fraction

**Bonus #1**: 10 pts: Complete, accurate, and justified responses (i: 3 pts; ii: 3 pts; iii: 4 pts)

**Bonus #2**: 5 pts: Exact-value calculation with complete and accurate justification.

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Gateway:_______/20  No Calc: _______/30  Calc: _______/50  Bonus: _______  TOTAL: _____/100