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Exercise for January 24, 2009

May 2003 Course 1 Examination, Problem No. 23, also Study Note P-09-08, Problem No. 71

The time \( T \) that a manufacturing system is out of operation has cumulative distribution function

\[
F(t) = \begin{cases} 
1 & - \left( \frac{2}{t} \right)^2, \quad \text{for } t > 2, \\
0 & \text{otherwise.}
\end{cases}
\]

The resulting cost to the company is \( Y = T^2 \). Determine the density function of \( Y \), for \( y > 4 \).

A. \( \frac{4}{y^2} \)  B. \( \frac{8}{y^3} \)  C. \( \frac{8}{y^3} \)  D. \( \frac{16}{y} \)  E. \( \frac{1024}{y^5} \)

Solution.
The cumulative distribution function of \( Y \) is

\[
F_Y(y) = \Pr(Y \leq y) = \Pr(T^2 \leq y) = \Pr(|T| \leq \sqrt{y}) = \Pr(T \leq \sqrt{y}) = F_T(\sqrt{y}) = 1 - \frac{4}{y},
\]
for \( y > 4 \). Differentiate to obtain the density function \( f_Y(y) = 4y^{-2} \). Alternatively, you can differentiate \( F(t) = F_T(t) \) to obtain \( f_T(t) = 8t^{-3} \) and consider the transformation given: \( y = t^2 \). The inverse of the transformation is \( t = \sqrt{y} \) and its derivative is

\[
\frac{dt}{dy} = \frac{1}{2\sqrt{y}}, \quad \text{so that}
\]

\[
f_Y(y) = f_T(t(y)) \left| \frac{dt}{dy} \right| = f_T(\sqrt{y}) \frac{d}{dy} \left( \sqrt{y} \right) = 8y^{-\frac{3}{2}} \cdot \frac{1}{2\sqrt{y}} = 4y^{-2}.
\]

Answer A.
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