May 2000 Course 1 Examination, Problem No. 18, Problem No. 19, also Study Note P-09-08, Problem No. 68
An insurance policy reimburses dental expense, \( X \), up to a maximum benefit of 250. The probability density function for \( X \) is:

\[
f(x) = \begin{cases} 
  ce^{-0.004x}, & \text{for } x \geq 0, \\
  0, & \text{otherwise},
\end{cases}
\]

where \( c \) is a constant. Calculate the median benefit for this policy.

A. 161  B. 165  C. 173  D. 182  E. 250

Solution.
We can see immediately that the form of the density is exactly as for the exponential distribution, and this requires that \( c = 0.004 \). Let \( R \) be the reimbursed amount and \( X \) be the actual expense. Then

\[
R = \begin{cases} 
  X, & X \leq 250, \\
  250, & X > 250.
\end{cases}
\]

The median benefit is the amount \( c \) such that \( \Pr(R \leq c) = 0.5 \). The median of the exponential distribution is always \( \ln 2 \) times its mean. Since

\[
\Pr(X > 250) = e^{-0.004 \times 250} = e^{-1} < 0.50,
\]

the median of \( R \) is the same as the median of \( X \), i.e., \( \frac{\ln 2}{0.004} \approx 173.29 \).

Answer C.