Exercise for January 31, 2009

**P Sample Exam Questions, Problem No. 138**

A machine consists of two components, whose lifetimes have the joint density function

\[ f_{X,Y}(x,y) = \begin{cases} 
\frac{1}{50} & \text{for } x > 0, y > 0, x + y < 10, \\
0 & \text{otherwise}. 
\end{cases} \]

The machine operates until both components fail. Calculate the expected operational time of the machine.

A. 1.7  B. 2.5  C. 3.3  D. 5.0  E. 6.7

Solution.
The joint density of \( X \) and \( Y \) is uniform over the region indicated by dotted lines in the figure below. In the figure, a line where \( y = x \) is also shown.
We are looking for $E(\max(X,Y))$. Note that $\max(X,Y) = X$ below the line $y = x$ and $\max(X,Y) = Y$ above that line. Note also that the lines $y = x$ and $x + y = 10$ intersect at the point $(5, 5)$. We have

$$E(\max(X,Y)) = E(\max(X,Y)|Y > X)\Pr(Y > X) + E(\max(X,Y)|Y \leq X)\Pr(Y \leq X) =$$

$$= E(Y|Y > X) \cdot \Pr(Y > X) + E(X|Y < X) \cdot \Pr(Y < X) =$$

$$= 5 \cdot \frac{1}{2} + 5 \cdot \frac{1}{2} = 5.$$

Answer D.

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